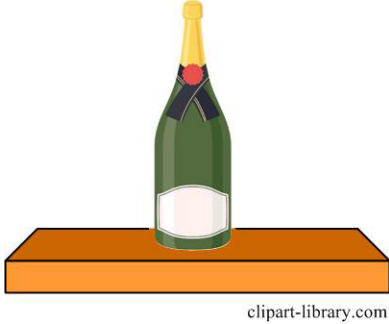


Knock the Bottle Problem

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This is a problem¹ from Barry Leung's Math Games.



Larry and Julius are playing a game, taking turns throwing a ball at a bottle sitting on a ledge. Larry throws first. The winner is the first person to knock the bottle off the ledge. At each turn the probability that a player knocks the bottle off the ledge is $\frac{1}{2}$, independently of what has happened before. What is the probability that Larry wins the game?

Solution

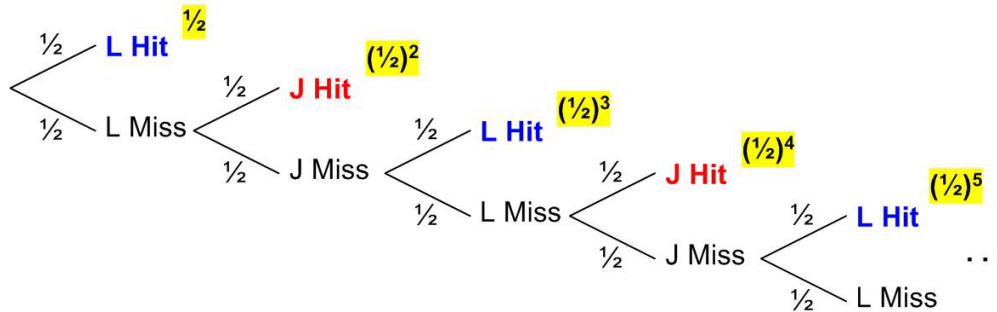


Figure 1

Figure 1 shows the sequence of events for the game with the associated probabilities of outcomes. When a player knocks the bottle off, the game stops, if not, the game continues. On the first play either Larry hits the bottle and knocks it off or he misses. The probability that Larry wins on the first play is $\frac{1}{2}$. If Larry misses, then Julius has a try. If he knocks the bottle off, he wins and the game stops. The probability of that happening is $(\frac{1}{2})^2$. This continues with the probability of a win at any stage highlighted in yellow. These outcomes are all independent of one another so they should all sum to 1.

Notice that the probabilities for a Larry win are all odd powers of $\frac{1}{2}$. So the overall probability for Larry to win is the sum of all these outcomes, namely

$$\sum_{k=0}^{\infty} \frac{1}{2^{2k+1}} = \frac{1}{2} \sum_{k=0}^{\infty} \frac{1}{4^k} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}.$$

Just as a check that everything adds to 1, the probability that Julius wins is the sum of the even powers of $\frac{1}{2}$, namely

$$\sum_{k=1}^{\infty} \frac{1}{2^{2k}} = \sum_{k=0}^{\infty} \frac{1}{4^k} - 1 = \frac{4}{3} - 1 = \frac{1}{3}.$$

And so the probability that Larry wins is $\frac{2}{3}$.

¹ 5 April 2026 (<https://medium.com/math-games/are-you-smarter-than-a-6-year-old-3e73146caded>)

Finite Game. Just for kicks we could consider what happens if we stop the game after, say, 4 chances. So each player had potentially the same number of opportunities to hit the bottle. What would be the probability that Larry would win in that case?

So the diagram in Figure 1 would stop after Julius's second try, at the most. Then Larry's probability of win is

$$\frac{1}{2} + \frac{1}{8} = \frac{10}{16}$$

and Julius's probability of win is

$$\frac{1}{4} + \frac{1}{16} = \frac{5}{16}$$

Notice that these probabilities do not add up to 1 but rather $^{15}/_{16}$. That is because there is a $^{1}/_{16}$ chance that neither player knocks the bottle off the ledge.

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