Integral Challenge

10 April 2026

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$$\int_2^4 f(t)dt = ?$$

This is a bit of a challenging problem from the 2026 Math Calendar ([1]).

Let f be a continuous real-valued function on the reals. For all t, $f(2t) = 3f(t)$ and $\int_0^1 f(t)\,dt = 1$. What is the value of $\int_2^4 f(t)\,dt$?

Again, the result must be a number of a day in a month.

Solution

First,

$$\int_0^1 f(2t)dt = 3\int_0^1 f(t)dt = 3$$

But making the substitution $s = 2t$ so $ds = 2dt$, we have

$$\int_0^1 f(2t)dt = \frac{1}{2}\int_0^2 f(s)ds = \frac{1}{2}\int_0^2 f(t)dt$$

implies

$$\int_0^2 f(t)dt = 6$$

Therefore,

$$\int_1^2 f(t)dt = 6 - \int_0^1 f(t)dt = 5$$

Again, set $s = 2t$ and $ds = 2dt$. Then

$$\int_1^2 f(2t)dt = \frac{1}{2}\int_2^4 f(s)ds = \frac{1}{2}\int_2^4 f(t)dt$$

But

$$\int_1^2 f(2t)dt = 3\int_1^2 f(t)dt = 15$$

So

$$\boxed{\int_2^4 f(t)dt = 2 \cdot 15 = 30}$$

References

[1] Rapoport, Rebecca and Dean Chung, *Your Daily Epsilon of Math*, *Wall Calendar 2026*, American Mathematical Society, 2026 May.