Root of the Problem

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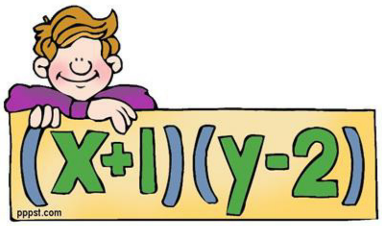
Jim Stevenson



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This is a problem[1] from Presh Talwalkar.

Given that x satisfies the equation:

$$x^4 + x^3 + x^2 + x + 1 = 0$$

What is the value of

$$(x^{33} + 2/x^{22})(x^{22} + 3/x^{33})$$

My Solution

The necessary idea, though somewhat esoteric, has come up before, most recently in the post "Two Algebra Problems".[2] When we see an expression like $x^4 + x^3 + x^2 + x + 1$, we should think of

$$x^5 – 1 = (x – 1)(x^4 + x^3 + x^2 + x + 1),$$

which give us the 5 roots of unity.[3] Therefore, $x^4 + x^3 + x^2 + x + 1 = 0$ means $x^5 = 1$. So $x^{33} = x^3\,(x^5)^6 = x^3$ and $x^{22} = x^2\,(x^5)^4 = x^2$ implies

$$(x^{33} + 2/x^{22})(x^{22} + 3/x^{33}) = (x^3 + 2/x^2)(x^2 + 3/x^3) = x^5 + 3 + 2 + 6/x^5 = 1 + 3 + 2 + 6 = 12$$

Talwalkar Solution

This is basically the same solution, only Talwalkar essentially derives the factorization for the 5 roots of unity.

$$x^4 + x^3 + x^2 + x + 1 = 0$$

Let us first ignore this equation and simplify the next expression:

$$(x^{33} + 2/x^{22})(x^{22} + 3/x^{33}) = x^{55} + 2 + 3 + 6/x^{55} = x^{55} + 5 + 6/x^{55}$$

If we can solve for x^{55} then we are done. But we also know 55 = 5×11, so we have:

$$x^{55} = (x^5)^{11}$$

Now all we need to do is solve for x^5, and we can evaluate the expression:

$$(x^5)^{11} + 5 + 6/(x^5)^{11}$$

Let's return to the given equation.

$$x^4 + x^3 + x^2 + x + 1 = 0$$

If we multiply both sides of the equation by x, the right hand side is 0, but the left hand side will have each exponent bumped up by 1.

$$x^5 + x^4 + x^3 + x^2 + x = 0$$

[1] February 26, 2026 (https://mindyourdecisions.com/blog/2026/02/26/only-smart-minds-see-it-solve-for-the-value/)
[2] November 1, 2025 (https://josmfs.net/wordpress/2025/11/01/two-algebra-puzzles/)
[3] Or think of the derivation of the partial sums of the geometric series $1 + x + x^2 + …$

Subtract the original equation from this to get:

$$x^5 - 1 = 0$$

$$x^5 = 1$$

Now we just substitute to get:

$$(x^5)^{11} + 5 + 6/(x^5)^{11} = 1^{11} + 5 + 6/1^{11} = 1 + 5 + 6 = 12$$

And the answer is 12. What an incredible puzzle!

References

1. https://www.facebook.com/photo.php?fbid=1204338798533743&set=pb.100068729014041.-2207520000&type=3
2. https://www.reddit.com/r/askmath/comments/1r8zkhz/does_anyone_know_how_to_solve_this_polynomial/