# Language Students Problem

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This is a problem from the 2001 American Invitational Mathematics Exam (AIME) ([1]).

Each of the 2000 students at a high school studies either Spanish or French, and some study both. The number who study Spanish is between 80 percent and 85 percent of the school population, and the number who study French is between 30 percent and 40 percent. Let $m$ be the smallest number of students who could study both languages, and let $M$ be the largest number of students who could study both languages. Find $M - m$.

## My Solution

Let $S$ be the fraction of students who study Spanish and $F$ the fraction who study French. Figure 1 shows the situation where the overlap consists of the students who study both Spanish and French.

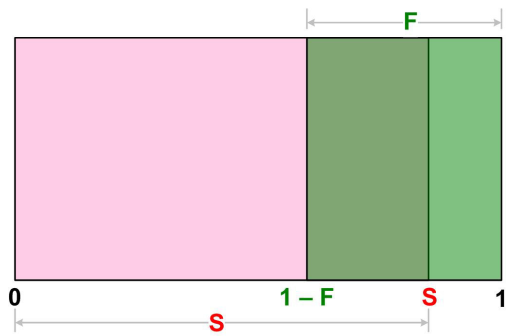


**Figure 1**

We are given that $0.8 \leq S \leq 0.85$ and $0.3 \leq F \leq 0.4$. So in terms of fractions of the student population $M$ is the maximum spread between $S$ and $1 - F$, and $m$ the minimum. Now

$$-0.3 \geq -F \geq -0.4 \;\Rightarrow\; 0.7 \geq 1 - F \geq 0.6$$

Therefore the maximum spread occurs when $S$ is maximal and $1 - F$ is minimal, or

$$M = 0.85 - 0.6 = 0.25$$

Similarly the minimal spread occurs when $S$ is minimal and $1 - F$ is maximal, or

$$m = 0.8 - 0.7 = 0.1.$$

Therefore

$$M - m = 0.25 - 0.1 = 0.15$$

as a fraction of the population, or

$$M - m = 0.15 \cdot 2000 = \text{300 students}$$

Another way to see this is to write the spread explicitly as $S - (1 - F) = S + F - 1$. Therefore,

$$M = 0.85 + 0.4 - 1 = 0.25 \text{ and } m = 0.8 + 0.3 - 1 = 0.1 \;\Rightarrow\; M - m = 0.15$$

## Comment

I changed the original problem to avoid an ambiguity AIME did not address, or at least address normally. Namely, AIME used 2001 as the total student population (for a problem in the year 2001), which meant the percentages did not result in whole numbers of students. Strangely, instead of rounding to the nearest integer, AIME truncated the values. Furthermore, they immediately applied the percentages to the population and used the truncated numbers of students in their max-min computations rather than fractions, thus arriving at 298 as their solution. Using 2001 instead of 2000 in my approach of fractions of population and converting that to students at the end of the problem resulted in an answer of 300.15 students, rounded to 300. I thought AIME's way of resolving the ambiguity of whole numbers was arbitrary and non-standard, and so made for a poor problem.

## References

[1] "Problem 2" 2001 AIME II Problems (https://artofproblemsolving.com/wiki/index.php/2001_AIME_II_Problems/)