Maximizing Love

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Jim Stevenson

This is a Valentine's Day puzzle[1] from BL's (Barry Leung) Math Games.

Happy Valentine's Day everyone, I hope you are having a euphoric moment, but if not, you can try this algebra puzzle about maximizing the expression LUV + LU + UV + LV given L + U + V = 12, where L, U, V are non-negative integers.

Solution

One approach relies on two past postings. The first idea[2] is to notice the L, U, V expressions are coefficients of the cubic polynomial where –L, –U, –V are the roots:

$$(x + \mathrm{L})(x + \mathrm{U})(x + \mathrm{V}) = x^3 + (\mathrm{L} + \mathrm{U} + \mathrm{V})x^2 + (\mathrm{LU} + \mathrm{UV} + \mathrm{LV})x + \mathrm{LUV}$$

So setting $x = 1$ and substituting L + U + V = 12, we have

$$(1 + \mathrm{L})(1 + \mathrm{U})(1 + \mathrm{V}) = 1 + 12 + (\mathrm{LU} + \mathrm{UV} + \mathrm{LV}) + \mathrm{LUV}.$$

Therefore we want to find the L, U, V that would maximize

$$(1 + \mathrm{L})(1 + \mathrm{U})(1 + \mathrm{V}) - 13 = (\mathrm{LU} + \mathrm{UV} + \mathrm{LV}) + \mathrm{LUV}.$$

where L + U + V = 12. For simplicity, let $x = \mathrm{L} + 1$, $y = \mathrm{U} + 1$, and $z = \mathrm{V} + 1$. Then the problem becomes maximize the function

$$f(x, y, z) = xyz - 13$$

subject to the constraint that

$$g(x, y, z) = x + y + z - 15 = 0.$$

This takes us to the second idea: Langrange multipliers. The current problem is actually a special case of the one addressed in the "Maximum Product" post,[3] which explains with diagrams the use of Lagrange multipliers in the optimization problem. The idea is that the maximum will occur when the gradients of the functions f and g are parallel, that is, when there is a constant λ (Lagrange multiplier) such that $\nabla f = \lambda \nabla g$.

Now $$\nabla f = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k}$$

and $$\nabla g = \mathbf{i} + \mathbf{j} + \mathbf{k}$$

So $$\nabla f = \lambda \nabla g \;\Rightarrow\; \lambda = yz = xz = xy \;\Rightarrow\; x = y = z.$$

Therefore $$0 = g(x, y, z) = 3x - 15$$

[1] https://medium.com/math-games/happy-valentines-day-can-you-maximize-the-love-on-this-day-807dfc6b1479
[2] "Challenging Sum" (https://josmfs.net/wordpress/2019/04/16/challenging-sum/)
[3] "Maximum Product" (https://josmfs.net/wordpress/2019/08/10/maximum-product/)

and $5 = x = y = z$

so the maximum is $f(x, y, z) = 5^3 - 13 = 112.$

Therefore the maximum of LUV + LU + UV + LV, given L + U + V = 12, is 112 when L = U = V = 4.

Comment. Barry Leung's solution begins similarly to mine and then disappears behind the subscription wall. But according to the comments, rather than calculus, Barry uses the arithmetic mean – geometric mean (AM-GM) inequality, which is also discussed in the "Maximum Product" post. From Wikipedia: [4]

AM-GM inequality states that for any set of [non-negative] real numbers their arithmetic mean is greater than or equal to their geometric mean.

$$(x_1 + x_2 + \ldots + x_n)/n \geq (x_1\, x_2 \ldots x_n)^{1/n}$$

In particular, equality is given if and only if $x_1 = x_2 = \ldots = x_n$.

I can never remember this formula, so I find the multivariable calculus solution easier! At my age I find it easer to remember procedures and derivations rather than formulas. I would be curious if this is common and if some psychologist has analyzed it.

[4] https://en.wikipedia.org/wiki/Inequality_of_arithmetic_and_geometric_means