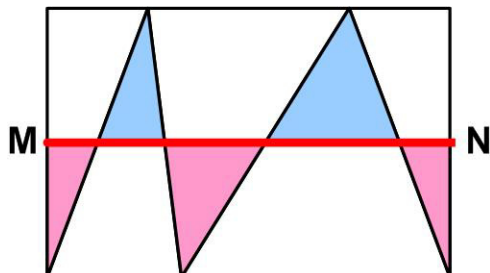


ZigZag in Rectangle

7 January 2026

Jim Stevenson



This is another problem¹ from BL's Math Games.

What fraction of the rectangle is colored? Assume that M and N are midpoints of the sides of the rectangle.

That they are midpoints was not stated explicitly in the problem as given in front of the subscription wall, but from the comments it became evident this was the case.

Initially I actually assumed the line was positioned arbitrarily. What would be the solution in that case?

Solution

Figure 1 shows what happens when the vertices of the triangles are moved parallel to their bases to the left side of the rectangle, preserving their areas. Because the red line splits the rectangle in half, the altitudes of the pink and blue triangles are the same. Therefore the reflected, blank images of the colored triangles are congruent to the corresponding colored triangles. So reflecting the blue triangles onto their blank congruent images we get Figure 2, which shows the colored area is one half of a half of the rectangle, or one quarter of the whole rectangle.

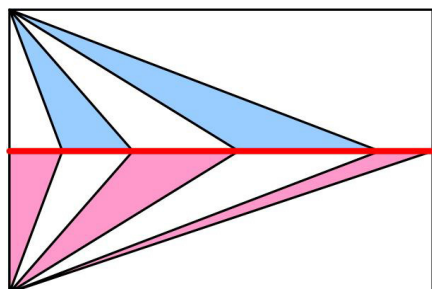


Figure 1

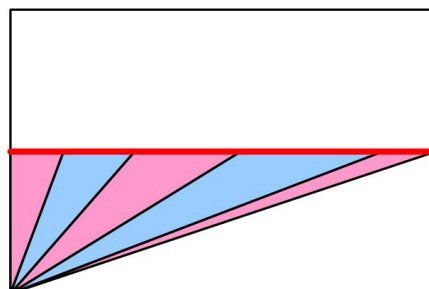


Figure 2

General Problem Solution

If the horizontal line were positioned arbitrarily, then following Polya's principle we could look at the extremes when it coincides with the top and bottom edges of the rectangle.

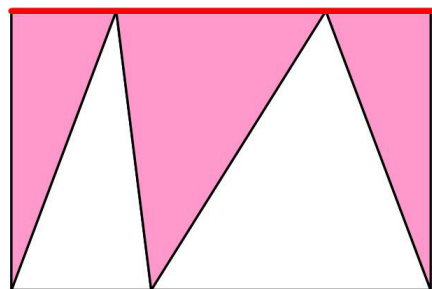


Figure 3

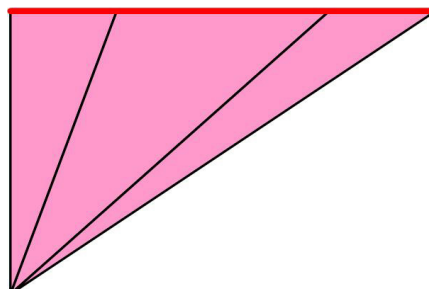


Figure 4

¹ 19 November 2025 (<https://medium.com/math-games/can-you-solve-this-in-6-or-7-seconds-f6c7b00e8caf>)

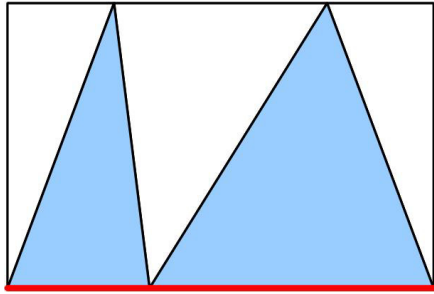


Figure 5

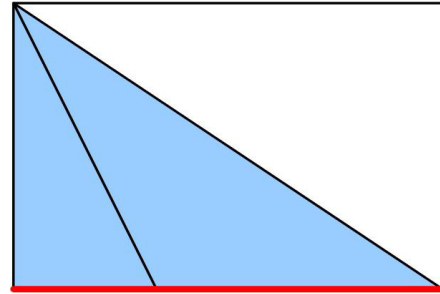


Figure 6

So we see from Figure 3 - Figure 6 that the colored areas for these extremes would be one half the area of the rectangle, and not one quarter. Initially, when I tried to solve the problem, I assumed the answer was one half. I then checked the midpoint case and found it was not a half. Upon reflection that made sense, since at the midpoint the uncolored trapezoids clearly have more area than the colored triangles.

Is One Quarter the Minimum Area?

Label the original diagram as in Figure 7. Then the area of the colored triangles is

$$A = \frac{1}{2} [h (r_1 + r_2 + r_3) + (W - h)(b_1 + b_2)]$$

Let $r = (r_1 + r_2 + r_3)$. Then $(b_1 + b_2) = L - r$. So the area becomes

$$A = \frac{1}{2} [h r + (W - h)(L - r)]. \quad (*)$$

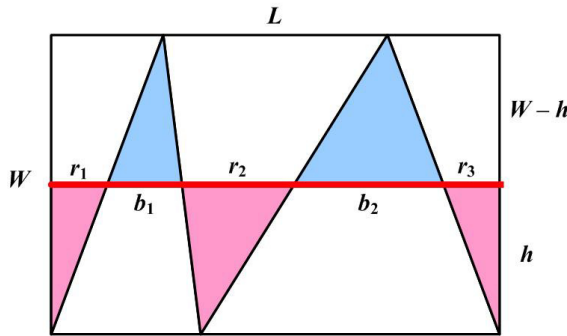


Figure 7

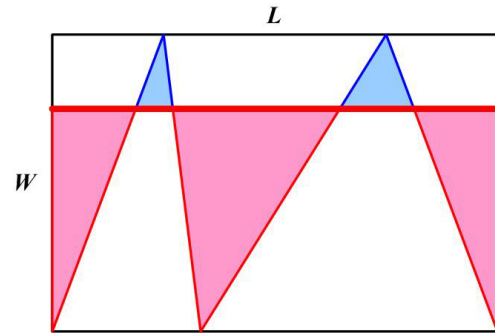


Figure 8

Consider what happens when the horizontal line moves up or down (Figure 8). By symmetric triangles (Figure 9) we have for $i = 1, 2, 3$, $r_i/h = r_i'/h'$. Therefore

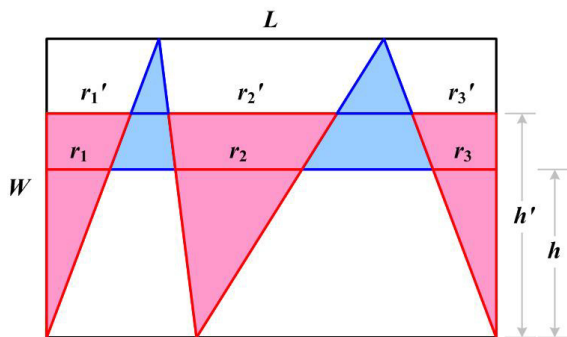


Figure 9

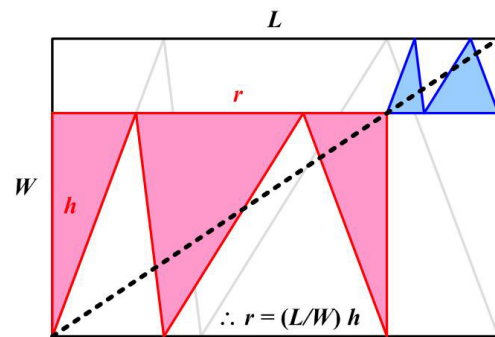


Figure 10

$$\frac{r}{h} = \frac{r_1 + r_2 + r_3}{h} = \frac{r_1}{h} + \frac{r_2}{h} + \frac{r_3}{h} = \frac{r'_1}{h'} + \frac{r'_2}{h'} + \frac{r'_3}{h'} = \frac{r'_1 + r'_2 + r'_3}{h'} = \frac{r'}{h'}$$

As the line varies from bottom to top, h' varies from 0 to W and r' varies correspondingly from 0 to L . (Of course, at $h' = 0$, $r' = 0$ the ratio is meaningless. But we are really interested in $h' > 0$.) So when $h' = W$, $r' = L$. Therefore we have

$$\frac{r}{h} = \frac{L}{W} \quad \text{or} \quad r = \frac{L}{W}h$$

This is shown graphically in Figure 10.

Therefore when the red triangles are slid to the left and the blue triangles to the right, the point of transition between the sum of their bases always lies on the diagonal of the original rectangle (Figure 10 and Figure 11). (I actually figured this out with Visio first and then worked out the geometric proof.)

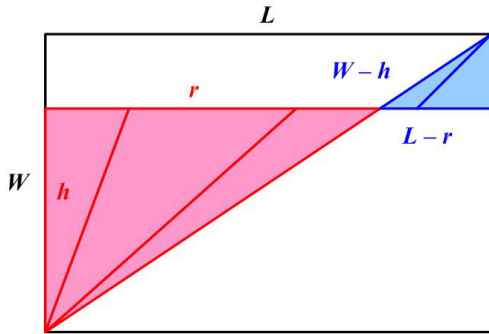


Figure 11

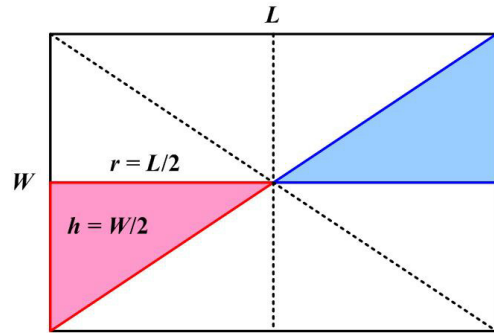


Figure 12

It is easy to see that when the horizontal line is at the midpoint of the rectangle, the rectangle consists of 8 right triangles congruent with the colored triangles (Figure 12). And therefore the 2 colored triangles make up $\frac{1}{4}$ of the area of the rectangle.

We still need to show the halfway point provides the minimal sum of areas. We resort to calculus. With the relationship $r = (L/W)h$, equation (*) becomes

$$\begin{aligned} A &= \frac{1}{2} [h(L/W)h + (W-h)(L - (L/W)h)] = \frac{1}{2}(L/W)[h^2 + (W-h)^2] \\ &= \frac{1}{2}(L/W)[2h^2 - 2Wh + W^2] \end{aligned}$$

Therefore $dA/dh = (L/W)[2h - W]$ and $d^2A/dh^2 = 2L/W > 0$.

The derivative $dA/dh = 0$ at only one point, $h = W/2$, and that is a minimum since the second derivative is positive. And so the area is minimal at the midpoint and equals a quarter of the area of the rectangle.

Comment. This means the general problem does not have a single answer, and in fact ranges from one half at the extremes to one quarter as the line moves to the halfway point. So by not stating explicitly that M and N are midpoints, the publicly presented problem is undetermined, that is, not well-defined. Of course it is not always easy to know what we can assume about a problem. We did assume it was a rectangle even though none of the angles were explicitly shown to be right angles. Catriona Agg's problems also make a lot of assumptions about the drawing, but she usually shows things like midpoints if that is the case.