

Three Buckets Question

3 October 2025

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From Presh Talwalkar¹ here is a variation of the three jugs problem.

You have buckets that hold 3 L, 7 L, and 20 L of water. How can you measure the following amounts?

- a) 10 L
- b) 4 L
- c) 5 L

For most of mathematical history the above information would be sufficient information to state the problem. But in today's society, there is a demand to state all assumptions as if that will make the problem better. So the test explained there are certain actions you can take.

You can fill any bucket completely with water. You can pour all the water from a bucket into a larger bucket. You can pour water from a bucket to fill a smaller bucket. You can empty the water completely from any bucket.

My Solution

In Talwalkar's original statement, he also included the sentence:

This question was posed to 8th grade students in the US, and only about 11 percent could correctly solve all parts. So nearly 90 percent missed this question.

That suggests the students should approach the solution via trial and error, which to me is largely tedious and boring. I generally dislike these types of problems, at least from a mathematical point of view. The fun, interesting, and elegant approach is to find a solution technique that works simply in all cases, and this is accomplished with the skew billiard table model (which I imagine is beyond the typical 8th grader).²

The skew billiard table setup is shown in Figure 1. Filling and emptying buckets is modeled by moving along the colored lines from one edge of the table to the opposite edge and bouncing like a billiard ball would. Namely if it hit a side at 60°, it would be reflected by the same angle of 60°. (One exception involves the corners with oblique angles: either of two exit directions can be taken.)

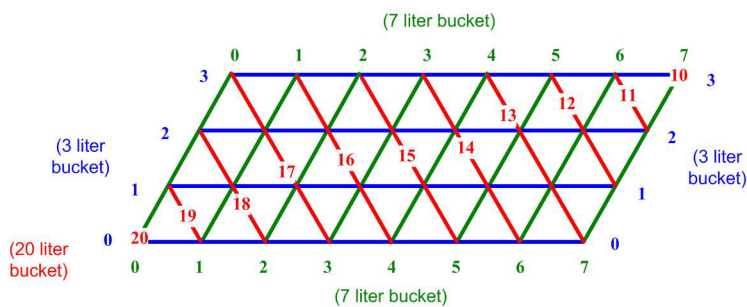


Figure 1

Moving along a horizontal blue line describes pouring water between the (red) 20L bucket and (green) 7L bucket and leaving the water in the (blue) 3L bucket alone. Moving along the red line

¹ <https://mindyourdecisions.com/blog/2025/09/22/3-bucket-question-90-percent-missed-this/>

² See "Three Jugs Problem" (<http://josmfs.net/2018/12/29/three-jugs-problem/>) for a description and explanation. A more streamlined example, as used here, is given in "Three Jugs Problem Redux" (<http://josmfs.net/2019/01/25/three-jugs-problem-redux/>).

describes leaving the water in the (red) 20L bucket alone and pouring water between the (blue) 3L bucket and (green) 7L bucket. And finally moving along the green line describes leaving water in the (green) 7L bucket alone and pouring water between the (blue) 3L bucket and (red) 20L bucket .

All my solutions start with a filled 20L bucket. (Most problems of this type involve no outside additions of water, so usually the largest container is filled first.) Clearly there are other starting points and there are always two paths (solutions) from the full 20L bucket to the desired amount (so long as there is at least one solution). Just trace the bounces back from the solution to the filled 20L bucket taking the other 60° direction.

a) 10 L

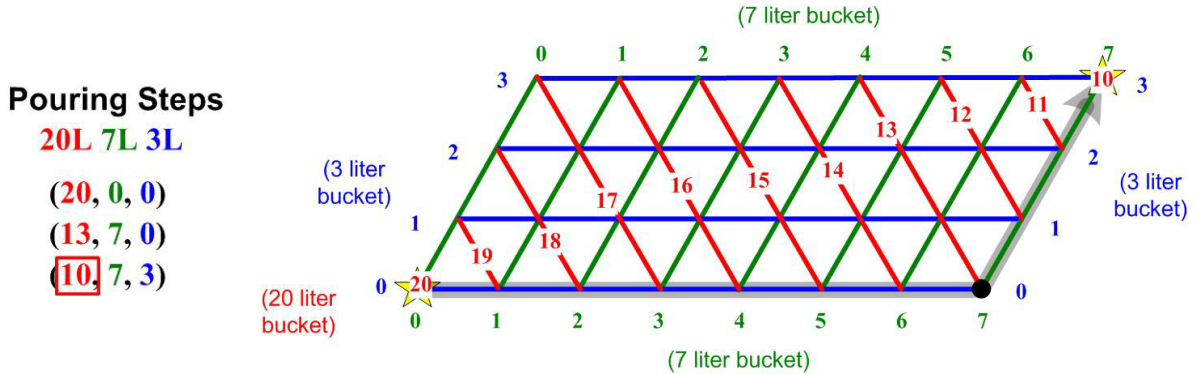


Figure 2

Follow the grey path along the billiard table (Figure 2). From a full 20L bucket, fill the 7L bucket (move horizontally to the right edge leaving the 3L bucket alone) leaving 13L in the 20L bucket. Again from the 20L bucket, fill the 3L bucket leaving 10L in the 20L bucket. And that is the solution.

b) 4 L

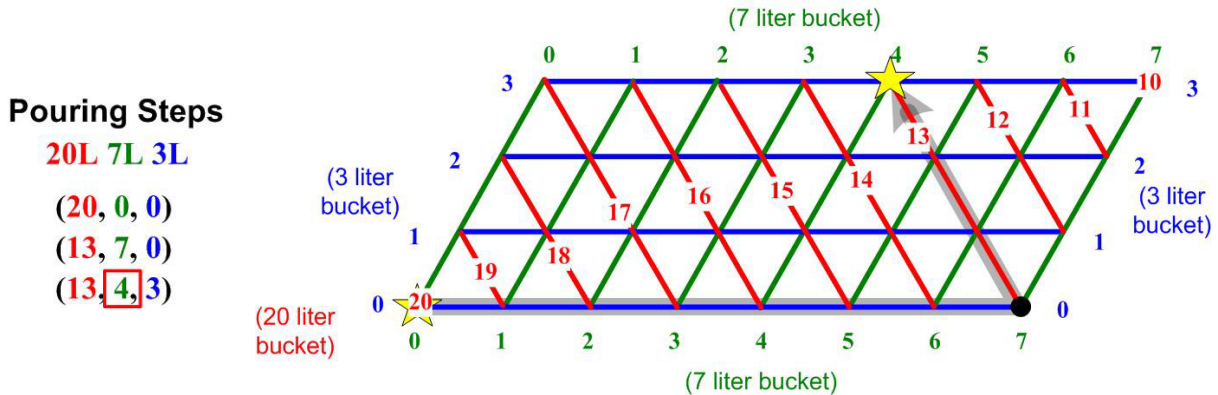


Figure 3

Following the grey path in Figure 3, again from a full 20L bucket, fill the 7L bucket (move horizontally to the right edge leaving the 3L bucket alone) leaving 13L in the 20L bucket. This time from the full 7L bucket, fill the 3L bucket, leaving 4L in the 7L bucket. And that is the solution.

c) 5 L

This case is more complicated as the path traced in the billiard table in Figure 4 indicates. How anyone could find this by trial and error is not clear. But the skew billiard table, once set up, can answer all the pouring questions, even the ones that are impossible (require solution nodes that are not along the edges of the table).

Pouring Steps

20L 7L 3L

(20, 0, 0)
 (13, 7, 0)
 (13, 4, 3)
 (16, 4, 0)
 (16, 1, 3)
 (19, 1, 0)
 (19, 0, 1)
 (12, 7, 1)
 (12, 5, 3)

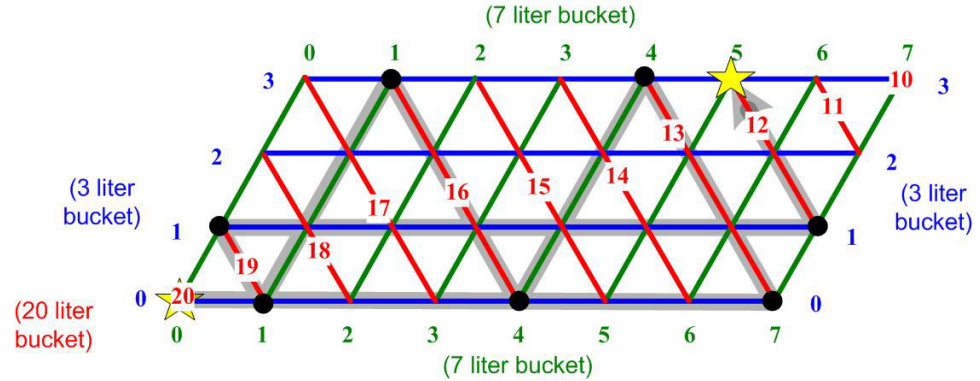


Figure 4

Talwalkar Solution

You have buckets that hold 3 L, 7 L, and 20 L of water. How can you measure the following amounts?

a) 10 L

Fill up the 3 L and 7 L buckets and transfer both to the 20 L bucket, which will have $3 + 7 = 10$ L.

b) 4 L

Fill the 7 L bucket and pour out water to completely fill the 3 L bucket. The remaining water is $7 - 3 = 4$ L.

c) 5 L

There are two main solutions.

Fill the 7 L bucket and pour out water to completely fill the 3 L bucket, leaving 4 L. Then transfer this 4 L to the 20 L bucket.

Empty the 3 L bucket. Then fill the 7 L bucket and pour to the 3 L bucket, leaving 4 L. Empty the 3 L bucket and again pour out from the 7 L bucket to fill the 3 L bucket. The remaining water will be $4 - 3 = 1$ L.

Pour the 1 L to the 4 L in the 20 L bucket, combining to a volume of $1 + 4 = 5$ L.

Another solution is to fill the 3 L bucket and pour it to the 20 L bucket. Repeat this so there will be 6 L, then 9 L, and finally 12 L in the 20 L bucket. Now pour out to completely fill the 7 L bucket, leaving $12 - 7 = 5$ L.

The problem has a connection to solving the equation below in integers:

$$7x + 3y = 5$$

The first solution requires filling the 7 L bucket 2 times and reducing the volume by the 3 L bucket 3 times, so we would have the equation:

$$7(2) + 3(-3) = 5$$

The second solution requires filling the 3 L bucket 4 times and reducing it by the volume of the 7 L bucket 1 time, so we have the equation:

$$7(-1) + 3(4) = 5$$

Extensions

I have covered similar problems in the past.

Perhaps students missed the question today, because it has been too long since the 1995 movie *Die Hard With A Vengeance* featured a similar problem! The task was to measure exactly 4 gallons from buckets that hold 3 gallons and 5 gallons.

Die Hard riddle

<https://www.youtube.com/watch?v=KfNRArPXCjw>

Another fun puzzle is to measure exactly 6 L from jugs of size 12 L, 8 L, and 5 L.

3 Jug Riddle

<https://mindyourdecisions.com/blog/2015/09/13/the-3-jug-riddle-sunday-puzzle/>

<https://www.youtube.com/watch?v=9fZB4s38Ygg>

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