Spot in a Rectangle Problem

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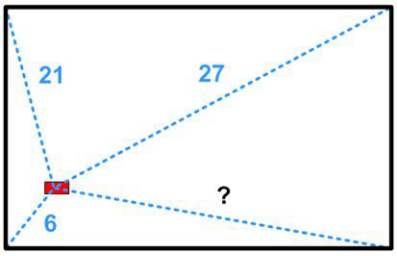

This puzzle is from the Irishman Owen O'Shea ([1]).

The following puzzle illustrates a beautiful mathematical relationship involving a rectangle of any size and a random point within that rectnangle that most people, including mathematicians, are unaware of.

The figure shows a rectangular room. There is a matchbox located 6 feet from one corner of the room and 27 feet from the opposite corner. The mathcbox is also loocated 21 feet from a third corner.

How far is the matchbox from the fourth corner?

Solution

Add the horizontal and vertical lines through the center of the matchbox as shown in Figure 1, and then label the segments x, y, u, v as shown. Label the unknown length z.

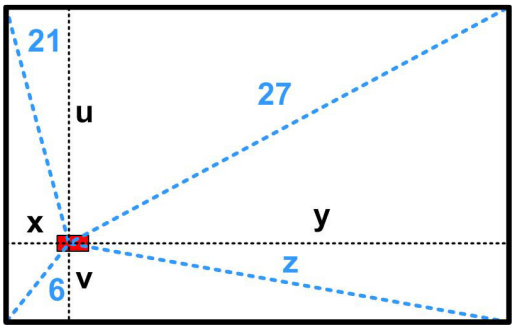

Figure 1

Then from the Pythagorean theorem we have the four equations

$$x^2 + u^2 = 21^2$$

$$x^2 + v^2 = 6^2$$

$$y^2 + u^2 = 27^2$$

$$y^2 + v^2 = z^2$$

Notice that if we subtract the second equation from the first and the fourth equation from the third, we get

$$u^2 - v^2 = 21^2 - 6^2 = 27^2 - z^2$$

or $$z^2 = 27^2 - 21^2 + 6^2 = (3\cdot 9)^2 - (3\cdot 7)^2 + (3\cdot 2)^2$$

or $$z^2 = 9((9 + 7)(9 - 7) + 4) = 9\cdot 36 = 9^2\cdot 4 = 18^2$$

So $$z = 18 \text{ feet}.$$

O'Shea's solution is essentially the same.

References

[1] O'Shea, Owen, *Mathematical Brainteasers with Surprising Solutions*, Prometheus Books, Guilford, Connecticut, 2020. But O'Shea got the problem from Martin Gardner, *The Colossal Book of Short Puzzles and Problems,* edited by Dana Richards, New York: Norton, 2006, problem 6.16, pp. 147, 159, 160.