

# Yet Another Track Puzzle

18 July 2023

Jim Stevenson

This is another problem from Dan Griller ([1]).

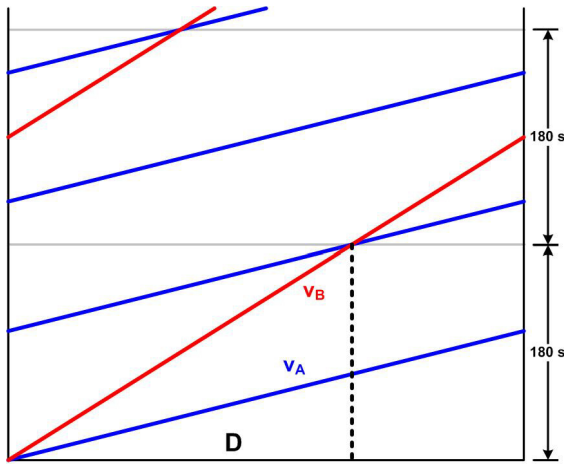


When Anthony and Benjamin run round a circular track in the same direction at constant but different speeds, they meet every 3 minutes. When Benjamin changes direction (but maintains his speed) they meet every 40 seconds.

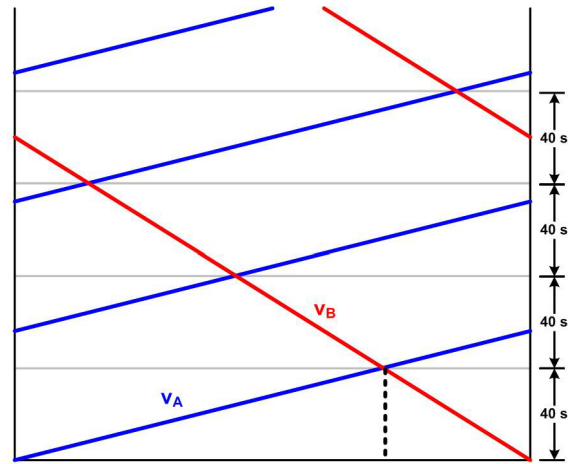
If Anthony is faster than Benjamin, calculate  
(speed of Anthony) / (speed of Benjamin)

## My Solution

Let  $v_A$  be Anthony's speed and  $v_B$  be Benjamin's speed. Then Figure 1 shows the case where Anthony is running in the same direction as Benjamin and Figure 2 shows the case where Benjamin is running in the opposite direction (not to scale).



1 Lap  
Figure 1



1 Lap  
Figure 2

Then from the first case we have

$$v_A 180 = 1 + D \quad \text{and} \quad v_B 180 = D$$

Then

$$v_A - v_B = 1/180$$

From the second case we get

$$v_A 40 + v_B 40 = 1 \quad \text{or} \quad v_A + v_B = 1/40$$

Let  $r$  be the ratio of the speeds  $v_A/v_B$ . Then we have

$$(r + 1) / (r - 1) = 180/40 = 9/2$$

Therefore

$$r = v_A/v_B = 11/7$$

## Griller Solution

Griller's solution is a bit abrupt, but essentially mirrors mine.

Let  $A$  be Anthony's speed and  $B$  be Benjamin's speed. Since 3 minutes is 180 seconds, the information given implies that

$$180(A - B) = 40(A + B)$$

This simplifies to  $7A = 11B$ , and so  $A/B = 11/7$ .

## References

- [1] Griller, Daniel, *Elastic Numbers: 108 Puzzles for the Serious Problem Solver*, Rational Falcon, 2017. Diamond Problem #24. (Scale of difficulty: Bronze, Silver, Gold, Diamond.)

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