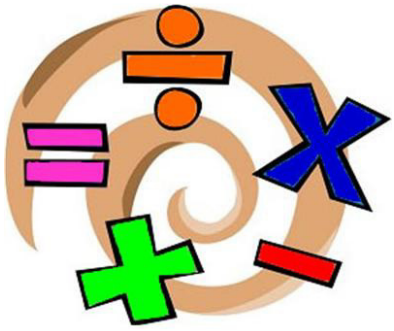
Two Radical Problems

31 October 2025

Jim Stevenson



clipart-library.com

Here are two problems involving those pesky radicals from the 2025 Math Calendar ([1]).

#1. Find the value of the expression

$$\sqrt{6+\sqrt{20}}-\sqrt{6-\sqrt{20}}$$

#2. Find the value of x in the expression

$$\sqrt{\log_{16}(x)+3}+\sqrt{\log_2(x)+12}=6$$

Incredibly the answers are days on the calendar.

Solution #1

If we square the expression, we get

$$\left(\sqrt{6+\sqrt{20}}-\sqrt{6-\sqrt{20}}\right)^2=\left(6+\sqrt{20}\right)-2\sqrt{36-20}+\left(6-\sqrt{20}\right)=12-8=4$$

So taking the positive square root, since the original expression is positive, we get

$$\sqrt{6+\sqrt{20}}-\sqrt{6-\sqrt{20}}=2.$$

Solution #2

Let $y = \log_{16}(x)$. Then $x = 16^y = 2^{4y}$. So $y = \frac{1}{4}\log_2(x)$. Therefore,

$$6=\sqrt{\log_{16}(x)+3}+\sqrt{\log_2(x)+12}=\tfrac{3}{2}\sqrt{\log_2(x)+12}$$

$$\log_2(x) + 12 = 4^2$$

$$\log_2(x) = 4$$

$$x = 2^4 = 16$$

References

[1] Rapoport, Rebecca and Dean Chung, *Mathematics 2025: Your Daily epsilon of Math*, American Mathematical Society, 2025. August