

Two Algebra Puzzles

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Here are two algebra problems from the 2025 Math Calendar ([1]).



#1. Find the value of the ratio

$$\frac{5^{18} - 5}{5^{16} + 5^{15} + \dots + 5^1 + 1}$$

#2. Find the value of the sum

$$z^{90} + z^{87} + \dots + z^3 + 1$$

where $z^2 + z + 1 = 0$.

Remember the answers are days on the calendar.

Solution #1

For slight simplicity let $x = 5$. Then the ratio becomes

$$\frac{x^{18} - x}{x^{16} + x^{15} + \dots + x^1 + 1} = \frac{x(x^{17} - 1)}{x^{16} + x^{15} + \dots + x^1 + 1} = \frac{x(x-1)(x^{16} + x^{15} + \dots + x^1 + 1)}{x^{16} + x^{15} + \dots + x^1 + 1} = x(x-1)$$

So

$$x(x-1) = 5 \cdot 4 = 20.$$

Solution #2

Let $y = z^3$. Then $z^{90} + z^{87} + \dots + z^3 + 1 = y^{30} + y^{29} + \dots + y + 1$. Now

$$y - 1 = z^3 - 1 = (z-1)(z^2 + z + 1) = 0.$$

Therefore $y = 1$ and the sum $z^{90} + z^{87} + \dots + z^3 + 1 = y^{30} + y^{29} + \dots + y + 1 = 31$

References

[1] Rapoport, Rebecca and Dean Chung, *Mathematics 2025: Your Daily epsilon of Math*, American Mathematical Society, 2025. June, May

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