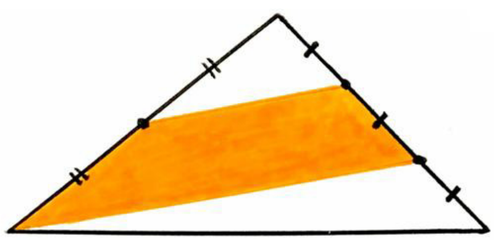
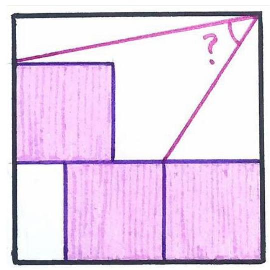
Geometric Puzzle Musings

16 July 2025

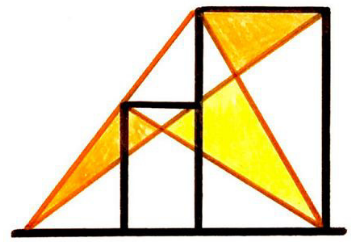
Jim Stevenson



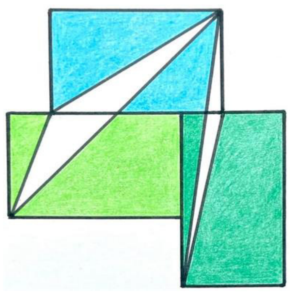
#1 What fraction of the triangle is shaded?



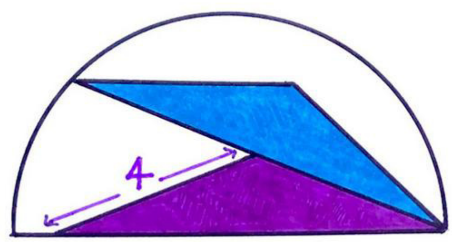
#2 The three smaller squares are the same size. What's the angle?



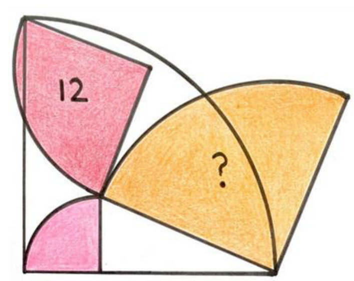
#3 The area of the yellow triangle is 15. What's the total orange area?



#4 The three rectangles are congruent. What fraction of the design is shaded?



#5 These triangles are congruent and isosceles. What's the area of the semicircle?



#6 Four quarter circles. What's the area of the orange one?

This is another collection of puzzles from Catriona Agg.

Solution #1[1]

Let T be the area of the enclosing triangle. Then from Figure 1 we see that the area A of the shaded region is

$$A = \tfrac{2}{3}\,T - \tfrac{1}{6}T = \tfrac{1}{2}\,T.$$

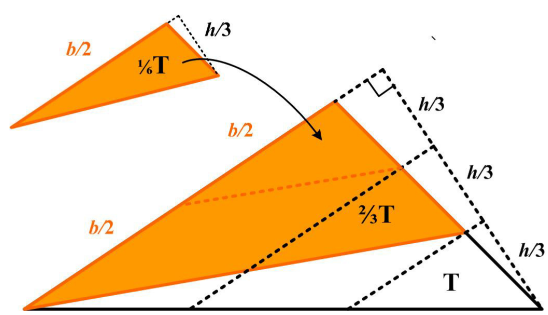


Figure 1

Solution #2[2]

Label the length of the edges of the small squares s and the remaining space on the sides of the large square x (Figure 2). Add the dashed pink line as shown. Then the pink lines in the figure define two (blue) right triangles that are congruent (SAS).

[1] May 29, 2025 at 9:20 AM (https://bsky.app/profile/catrionaagg.bsky.social/post/3lqcsphprac2d)

[2] February 25, 2024 (https://www.instagram.com/cshearer41/p/C3yMEEgtNZ0/)

The right blue triangle is then a 90º rotated version of the left blue triangle, so that the pink lines make an isosceles right triangle (Figure 3). That means the base angles are 45º and we are done.

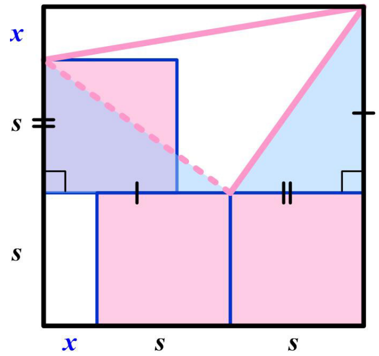


Figure 2

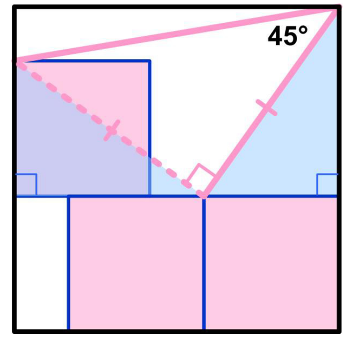


Figure 3

My Solution #3[3]

Let the first orange-outlined triangle be "orange triangles + white space" and the second orange-outlined triangle be "yellow triangle + white space." Then Figure 4 and Figure 5 show these two triangles have the same area as the congruent blue triangles.[4] Subtracting the common white area means the areas of the orange triangles and the yellow triangle are the same, namely 15.

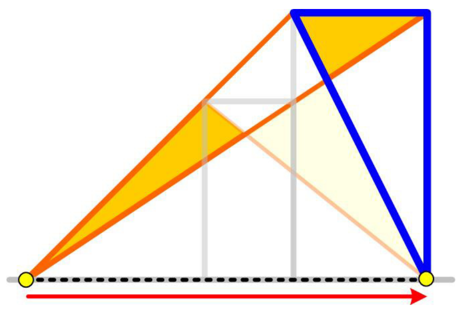


Figure 4

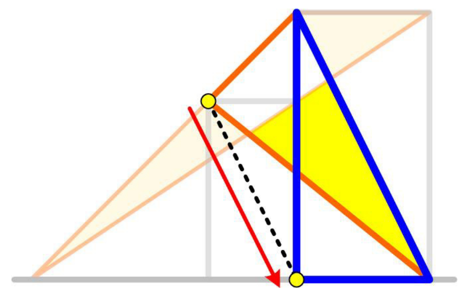


Figure 5

BL Solution

This is an older Catriona Agg problem that I found at BL's Math Games.[5] BL introduces the problem somewhat irritatingly, but par for the course for some online math problems.

Perhaps the UK government can use this puzzle instead to find innovators and disruptors to work for them instead. Except the requirement would be if the candidate can spot the solution in 1 second. Okay, I get it. 1 second is a bit too harsh. Let's say 10 seconds then. But yeah the solution is literally one line and that's it. Good luck!

[Figure 6] shows the line I am talking about. In case you can't see [in Figure 7] each orange triangle has the same area as its corresponding yellow triangle, I have added a few lines to indicate their base and height.[6]

3 Oct 29, 2020 (https://x.com/Cshearer41/status/1321738616482254849)

4 JOS: Just to be clear. The parallelism in the second figure comes from the fact that the two rectangles are similar due to their intersections with the lines of the orange triangles.

5 14 July 2025 (https://medium.com/math-games/i-solved-this-geometry-puzzle-in-literally-1-second-6ec18ed187d1)

6 JOS: And I have added some arrows outside the diagram as another way to see the equivalences.

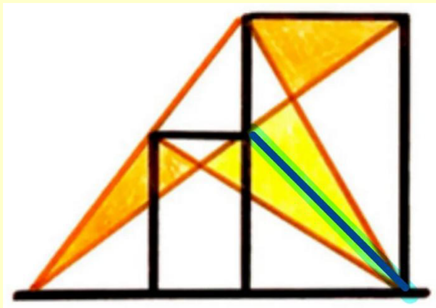


Figure 6

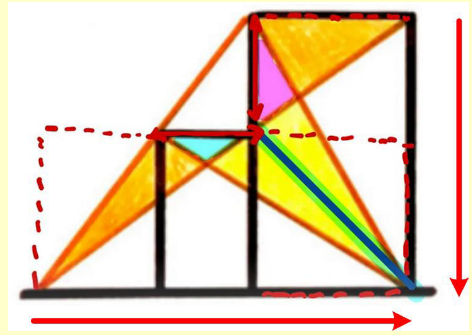


Figure 7

Solution #4[7]

The solution is shown in Figure 8. The vertices of the two triangles are shifted parallel to their bases to the common point on the blue rectangle. Since the altitudes of the triangles are preserved, the areas of the resulting triangles are the same as the originals.

From the figure we can see that the triangles leave half of the blue rectangle uncovered. Therefore, the shaded (colored) regions of the three rectangles make up 5/6 of the total area.

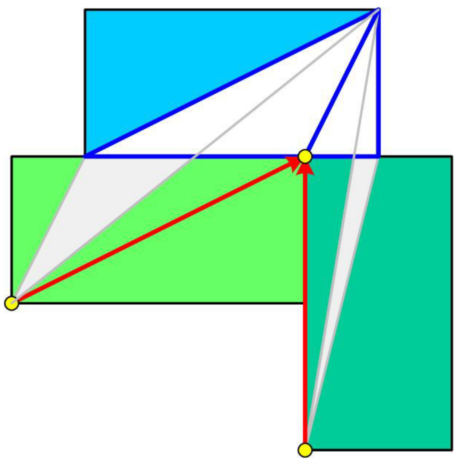


Figure 8

Solution #5[8]

The perpendicular bisector of the base of the blue triangle passes through the center of the semicircle. Since the congruent blue and purple triangles are isosceles, they have equal base angles. Therefore the black outlined right triangles are similar. Since they have a side in common, that means they are congruent. Therefore the radius of the semicircle is 4, and that means the area of the semicircle is

$$\frac{1}{2} \pi 4^2 = 8 \pi.$$

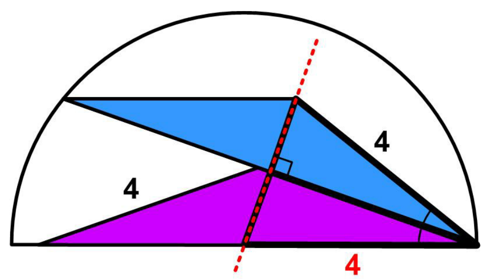


Figure 9

Solution #6[9]

It is easier to understand the problem if we replace the quadrants of the circles with squares. (I made the assumption that the radii of the red and orange quadrants were parallel.) Multiplying the areas of the squares by the constant factor π/4 yields the areas of the quadrants.

The ultimate simplicity of the solution belies the effort it took me to find it. First, I couldn't tell if the area 12 referred just to the mid-sized quadrant or to both the mid-sized and the smallest quadrant,

---

[7] July 5, 2025 at 7:03 AM (https://bsky.app/profile/catrionaagg.bsky.social/post/3lt7mbgldxs2s)

[8] June 21, 2025 at 6:05 AM (https://bsky.app/profile/catrionaagg.bsky.social/post/3ls4cjjrvuk2f)

[9] June 28, 2025 at 1:46 AM (https://bsky.app/profile/catrionaagg.bsky.social/post/3lsnhbfsjrk2d)

since the colors looked the same. A simpler problem would be if the area only referred to the mid-sized quadrant, so I made that assumption.

As I tried to construct the figure in Visio, I realized the vertex of the red square lay on the semicircle with diameter the diagonal of the largest black square. That meant the size of the orange square was determined by the red square—a good sign. But then it seemed that multiple configurations for different sized red squares would satisfy the problem, making it under-determined. But then I saw the purpose of the smallest square without an area: it forced the coinciding corners of the red and orange squares to lie on the other diagonal of the large black square, thus determining only one configuration. (The purpose of this exercise, other than enabling me to draw the figure, is to find out what is essential in the problem and so must figure in the solution.)

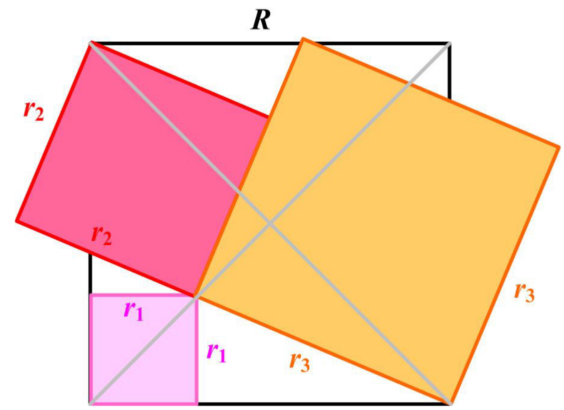


Figure 10

I labeled the sides of the squares, which were the radii of the quadrants (Figure 10). I then began to construct a bevy of equations using the Pythagorean Theorem. This was getting very complicated and it didn't seem like something Catriona Agg would do. Besides, I wasn't finding enough equations to resolve all the variables (sides of the squares).

Then, constructing one more line became the key: I added the diagonal of the red square to the mix (Figure 11).

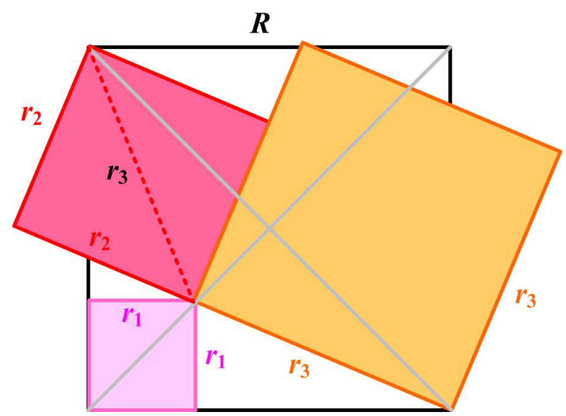


Figure 11

We see immediately that the common corners of the two squares lying on the diagonal, which is a perpendicular bisector of the other diagonal, means the corners are equidistant from the other two corners, and so

$$r_3^2 = 2\, r_2^2$$

and $$(\pi/4)r_3^2 = 2\,(\pi/4)r_2^2 = 2 \cdot 12 = 24$$

So the area of the orange quadrant is 24. Easy, *once you see it*!

Comment. I thought I would give this lengthy discussion of how I got the solution to show once more that a simple answer is not necessarily easy to find. Generally, problems are solved by circuitous routes. Often once a solution is found, a more direct route becomes evident, erasing the wanderings that seem a bit embarrassing in retrospect. Finding the simple solution immediately is the ultimate high in solving these problems.