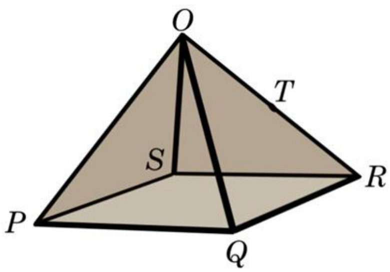
# Shortest Pyramid Path

2 February 2025

Jim Stevenson




This is an earlier puzzle[1] from Presh Talwalkar.

A square pyramid has base *PQRS* and vertex *O*. Each edge has length equal to 20. Calculate the shortest distance along the outer surface of the pyramid from *P* to *T*, the midpoint of *OR*.

## My Solution

The idea is to turn the pyramid into an equivalent flat surface on which straight lines mark the shortest distances between two points.

Figure 1 shows such a flattening where we have repeated the two faces opposite the point P whose edges contain the point T. The diagram shows all the possible attachments of the faces containing T to the other faces. By symmetry we only need consider two of the four images of point T.

We draw a circle centered on P and passing through the nearest point T. The radius of this circle gives the minimal distance to T. Since the faces are equilateral triangles, their altitude is $\frac{1}{2}\sqrt{3}$ times their side and so $10\sqrt{3}$. Therefore the radius squared is $10^2(2^2 + 3)$, so the radius is $10\sqrt{7}$.

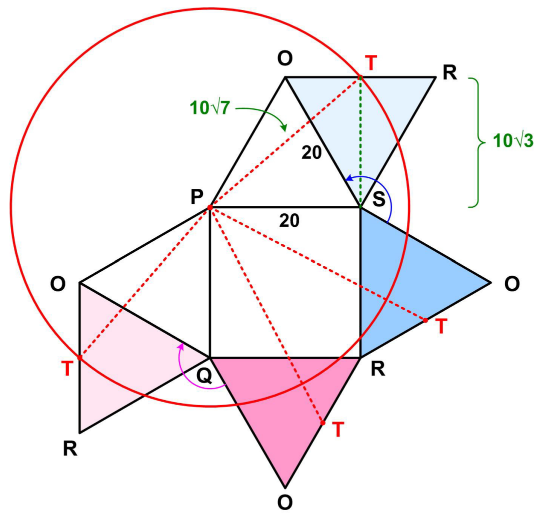



**Figure 1**

## Talwalkar Solution

There are two paths we need to check: one path along the base of the pyramid and the other along the faces. We will calculate the length of each path and take the shorter length (Figure 2).

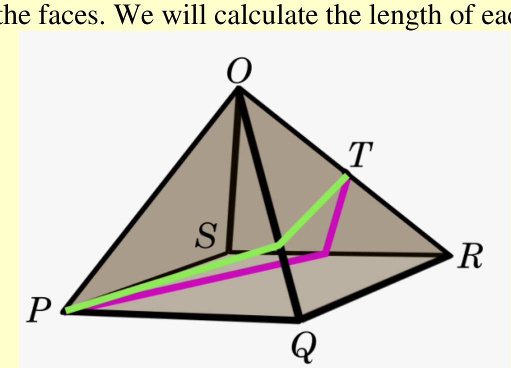



**Figure 2**

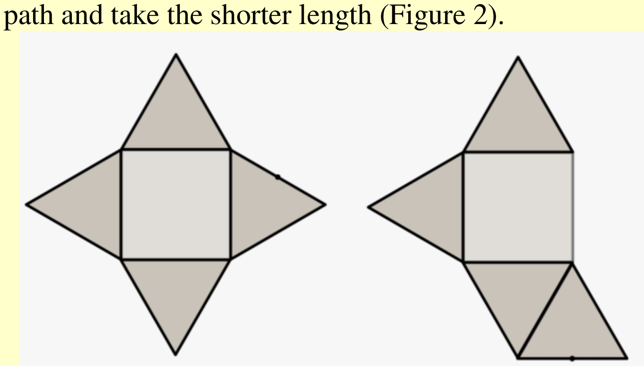


**Figure 3**

[1] 13 September 2022 (https://mindyourdecisions.com/blog/2022/09/13/shortest-path-around-a-pyramid/)

We will calculate the length of each path by unfolding the pyramid into a net (Figure 3). Then the path from $P$ to $T$ will be a straight line segment and calculated easily.

**Path along square base**

We know $RT = 10$, and each triangular face is an equilateral triangle with each angle equal to 60 degrees. Thus the horizontal distance from $T$ to the triangular base is $10 \sin(60°) = 5\sqrt{3}$ and the other leg of the triangle is $10 \cos(60°) = 5$.

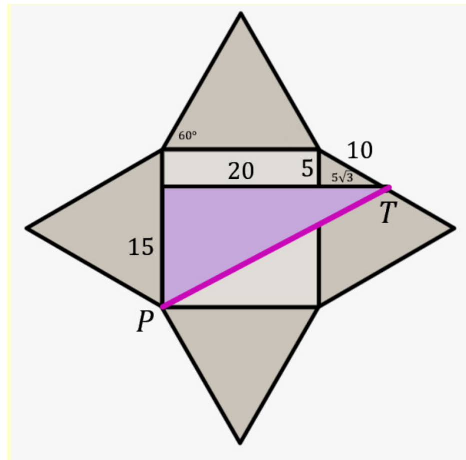


**Figure 4**

Each side of the square base has length equal to 20, so the legs of the triangle with hypotenuse $PT$ have lengths of $20 - 5 = 15$ and $20 + 5\sqrt{3}$. Thus we have:

$$\text{length from } P \text{ to } T = \sqrt{15^2 + (20 + 5\sqrt{3})^2}$$
$$= \sqrt{225 + 400 + 200\sqrt{3} + 75}$$
$$= 10\sqrt{7 + 2\sqrt{3}}$$
$$\approx 32.348$$

**Path along triangular faces**

The net of this pyramid looks like this in Figure 5

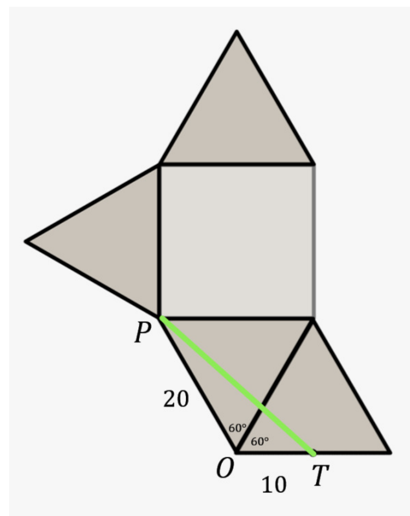


**Figure 5**

We know $PO = 20$ and $OT = 10$, and the angle $POT = 60° + 60° = 120°$. Thus we can find $PT$ by Al-Kashi's law of cosines.

$$\text{length from } P \text{ to } T = \sqrt{20^2 + 10^2 - 2(20)(10)\cos(120°)}$$
$$= \sqrt{400 + 100 + 200}$$
$$= 10\sqrt{7}$$
$$\approx 26.458$$

This path has a shorter length, and thus this is the distance from $P$ to $T$. The answer is $10\sqrt{7} \approx 26.458$.

**Source**

Math StackExchange:

https://math.stackexchange.com/questions/3289765/shortest-distance-around-a-pyramid

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