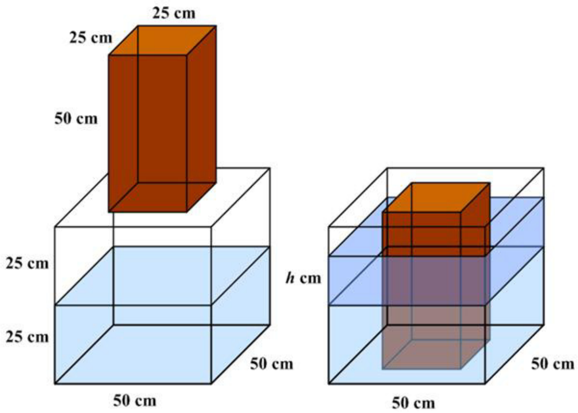
Brick in Water Puzzle

25 April 2025

Jim Stevenson



I thought this puzzle, which was included among a set of seven challenges[1] assembled by Presh Talwalkar, would be fairly straight-forward.

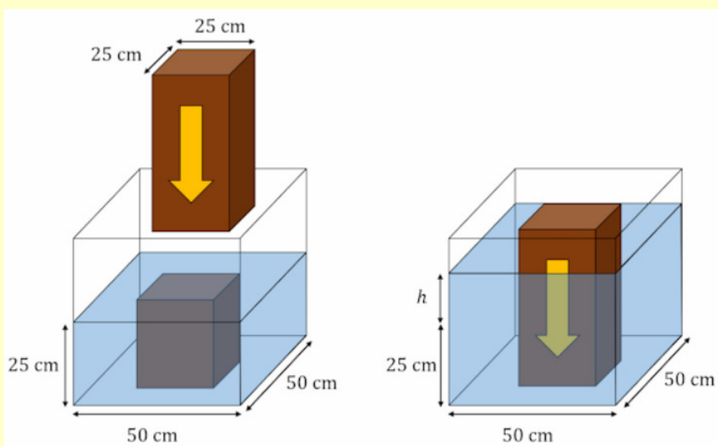
A cube of 50 cm is filled halfway with water. A rectangular prism with a square base of 25 cm and a height around 50 cm is placed flat onto the base of the cube, as shown. By how much does the water level rise?

Thanks to Fahad Alomaim for the suggestion! This is translated from a Mawhiba curriculum question for 8th grade.

But I got the wrong answer and found Talwalkar's solution a bit hard to fathom at first. Looks like I flunked 8th grade.

Talwalkar's Solution

Let h be the increase in the height of the water level. The amount of water displaced is equal to the volume of a cube with a side length of 25 cm.[2]

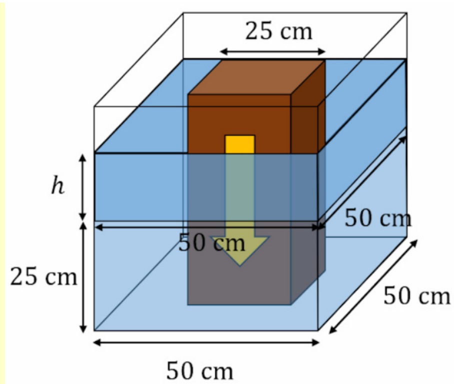


$$25 \times 25 \times 25 = 15625 \text{ cm}^3$$

This volume is distributed into a new shape, which is a rectangular prism with a square base of 50 cm minus that of a square base of 25 cm. The height is h.

[1] 16 April 2025 (https://mindyourdecisions.com/blog/2025/04/16/7-challenging-logical-puzzles)

[2] JOS: This was not obvious to me. In fact I originally made a different assumption. See my solution below.



So we have:

JOS: Don't multiply out the numbers! Keep powers of 25.

$$15625 = 50^2h - 25^2h$$

$$15625 = h(50^2 - 25^2)$$

$$15625 = h(1875)$$

$$h = 8.333\ldots \text{ cm}$$

My Solution

I assumed the brick displaces a volume of $50 \times 50 \times (25 + h)$ cm^3. So this must be the volume of water that raises the height of the water in the cube by h cm.[3] Therefore we have the equation

(Increased volume of water) = (volume displaced by brick)

or $$h(50^2 - 25^2) = (25 + h)25^2$$

or $$4{\cdot}25^2\, h - 25^2\, h = 25^3 + 25^2\, h$$

So $$2h = 25$$

or $$h = 12.5 \text{ cm}$$

which does not agree with Talwalkar's answer.

Alternative Approach.

But after seeing Talwalkar's solution I tried to reason differently.

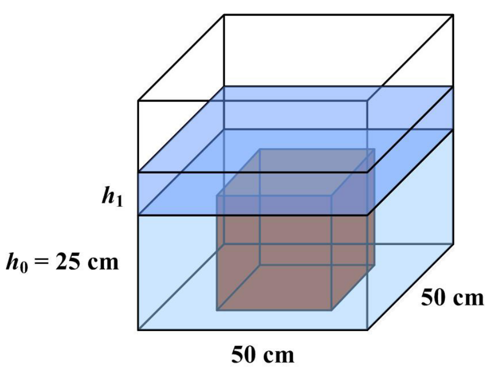


Figure 1

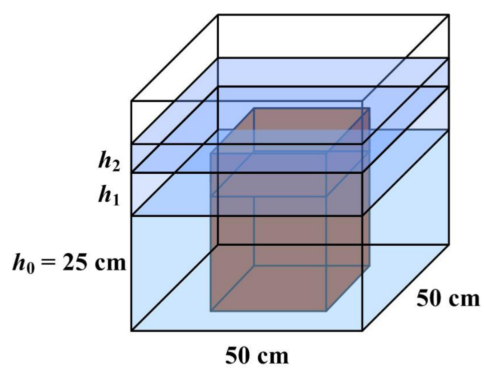


Figure 2

[3] JOS: This assumption turned out to be false.

Suppose the brick is a cube 25 cm on an edge. That would mean the increased height h_1 would be the thickness of a layer of water above the brick as shown in Figure 1, namely,

$$h_1\, 50^2 = 25^3 = h_0\, 25^2$$

So

$$h_1 = ¼\, h_0$$

Now we sort of have a new, identical problem of adding a brick to the bottom of a layer of water in the cube. So add an extension to the brick of thickness h_1 cm. This would mimic the original addition of the brick as a cube, and this would raise a layer of water h_2 cm thick above the extension (Figure 2). The volume of this layer would be

$$h_2\, 50^2 = h_1\, 25^2$$

and

$$h_2 = ¼\, h_1 = (¼)^2\, h_0$$

Continuing in this way we would have an infinite sequence of incremental heights, $h_n = ¼\, h_{n\text{-}1}$ with $h_n \rightarrow 0$, whose sum would be

$$h = h_0\,[¼ + (¼)^2 + (¼)^3 + (¼)^4 + \ldots] = h_0\,[¼ / (1 - ¼)] = h_0 / 3 = 25 / 3 = 8.3333 \text{ cm}$$

which is Talwalkar's answer! As the brick grows above this height there would be no further displacement of water.

Weird. So the brick only displaces the original 25cm cube of water. I can't quite get my head around this.

Yet Another Approach.

Okay, now I think I get it. Just imagine the brick touching the top of the water. As it lowers, it just displaces the 25 cm cube of water that was there, and that is what gets pushed up around the sides of the brick. There is no other source of water. I suppose that is what Talwalkar was getting at from the start. I just didn't see it. (And it still feels just a tad counter-intuitive.)
