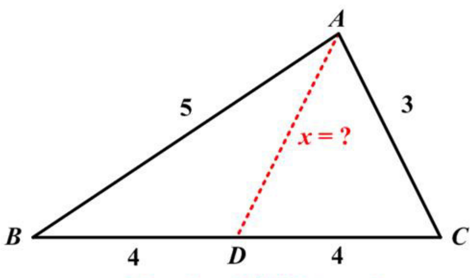
Tricky Triangle

19 February 2025

Jim Stevenson



I think this turned out to be an even trickier problem[1] than Alex Bellos thought.

Tricky triangle This one was sent in by a reader, aged 85, who first saw it in 1960. He is a roboticist who passed through Harvard, Princeton, Stanford and IBM. He says it is his favourite puzzle. 'I've given this puzzle to perhaps 100 people. Over 80% have no idea how to solve it.' What is the length of AD, the dashed line?

This time I am going to give Bellos's solution first (which I got, after first making the mistake he mentions).

Bellos Solution

Answer: x = 1

The reason that most people fail to solve this one is because the triangle is degenerate: it is, actually, a line, not a triangle. The horizontal side has length 8. The other two sides add up to 8, so these two sides will lie on top of the horizontal line.

People who spot that the triangle is degenerate still get the puzzle wrong, as they immediately assume that the distance between A and D is 0. A moment's reflection, and you realize it is 1.

My Solution

Yes, that is one interpretation. However, if by "not to scale" can mean lines BA and AC in fact form a straight line, then we could also consider what happens if lines BD and DC do *not* form a straight line. We are then in the realm of an ellipse, that is, the locus of all points the sum of whose distances to the points B and C (foci) is 8. (I feel further justified in considering this situation if I assume Bellos added the label "Tricky Triangle" and the originator did not mention a triangle.)

Figure 1 shows the Bellos solution. But it can also be interpreted as a degenerate ellipse with semimajor axis $a = 4$ and semiminor axis $b = 0$ (with distance between a focus and the center $c = 4$).

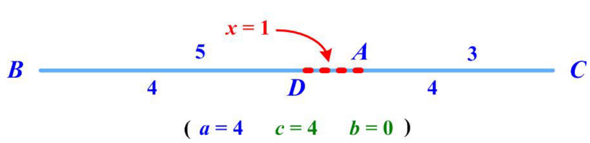


Figure 1

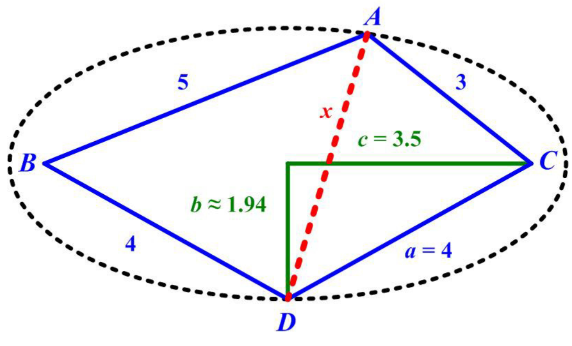


Figure 2

[1] 17 February 2025 (https://www.theguardian.com/science/2025/feb/17/can-you-solve-it-the-simple-geometry-question-that-fools-almost-everyone)

Figure 2 shows the case where $a = 4$ and $c = 3.5$. Then $b = ½\sqrt{15} \approx 1.94$.

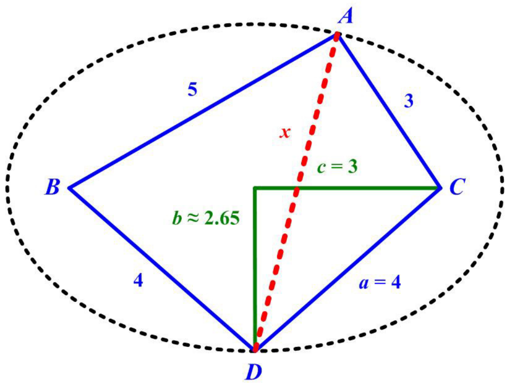


Figure 3

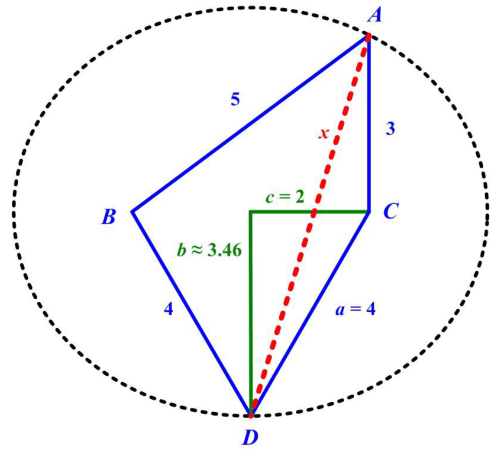


Figure 4

Figure 3 and Figure 4 show successively smaller distances between the foci and the center of the ellipses, namely, $c = 3$ and $c = 2$, respectively. In all these cases we have $1 \le x < 7$, since x is the base of a triangle with sides 3 and 4.

Figure 5 shows the case where x achieves its maximum value of 7. This occurs when $c \approx 1.5$.[2] For $c < 1.5$, we again see $1 \le x < 7$.

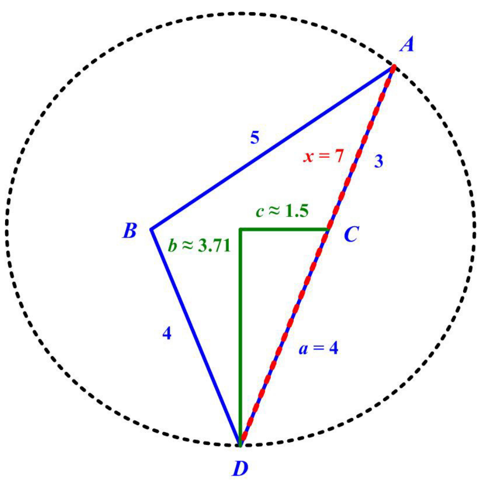


Figure 5

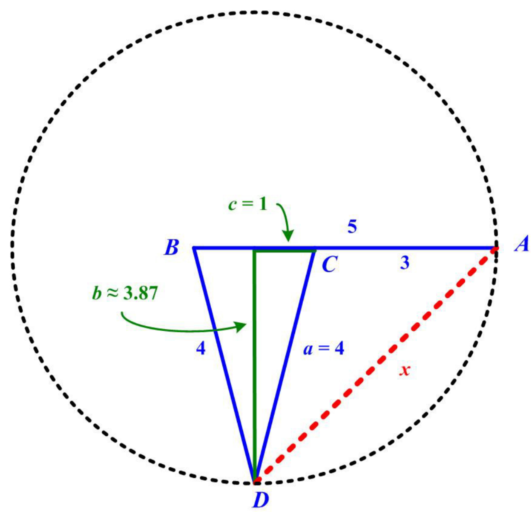


Figure 6

Since point A on the ellipse must always be 5 units from B, we must have $a + c \ge AB = 5$ or $c \ge 1$, since we always have $a = 4$. Figure 6 shows this limiting case where $c = 1$.

Therefore, we conclude that the general answer to the problem is $1 \le x \le 7$, depending on how one interprets the degeneracy of the figure.

(I drew all the figures to scale using Visio, which was a fun and challenging exercise in itself.)

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[2] The actual calculation is as follows. Using the parametric equation for an ellipse

$$r = \frac{p}{1 + e\cos\theta}$$

where $p = a(1 - e^2)$ and $e = c/a$. We know $a = 4$, and at x maximum, $\cos\theta = c/a = e = c/4$, and $r = 3$. Substituting these values yields $c = 4/\sqrt{7} \approx 1.5119$.