Noah and Population Growth

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My cousin sent me this query[1] from the dubious[2] Quora:

"In the Book of Genesis, only 8 humans, Noah and his sons and their four wives, survived the Flood. How many people could a family of 8 procreate in, say, 500 years?"

Solution

Crude Approximation

We are going to make a lot of "spherical cow"[3] assumptions to get an idea of what might happen. Suppose each generation is 25 years. Then 500 years represents 20 generations. Suppose each couple will have *m* number of children per generation, and suppose all the children instantly appear after those 25 years and the parents disappear, so that the population consists only of those children. Further suppose each generation of children is evenly divided between male and female so that we get *m*/2 couples per generation (so for now we assume *m* is even: 2, 4, 6, 8, 10, …). Early on we will have to accept that some incest between brothers and sisters might occur and certainly first cousins can marry. But since that problem began with Adam and Eve's children, it must be acceptable for Noah's progeny.

Then we have the following developments:

Table 1 Crude Population Approximation

Generation	Children (Couples × Children per Couple)	Couples (Children/2)	Population (2×Couples)
gen 0		$8/2 = 4$	8
gen 1	$4 \times m$	$4m/2$	$8m/2$
gen 2	$(4m/2) \times m$	$4(m/2)^2$	$8(m/2)^2$
gen 3	$4(m/2)^2 \times m$	$4(m/2)^3$	$8(m/2)^3$
…			
gen *n*	$4(m/2)^{n-1} \times m$	$4(m/2)^n$	$8(m/2)^n$

So we are interested in generation 20. Then the population will be

$$8(m/2)^{20} = 8\,m^{20} / 2^{20}$$

Now $2^{10} = 1024 \approx 1000 = 10^3$. So

$$\text{gen 20 population} = 8\,m^{20} / 2^{20} = 8\,m^{20} / (2^{10})^2 \approx 8\,m^{20} / (10^3)^2 = 8\,m^{20} / 10^6 = 8\times 10^{20\log m - 6}$$

So we make a new table of the possible population values after 500 years depending on the

[1] https://www.quora.com/In-the-Book-of-Genesis-only-8-humans-Noah-and-his-sons-and-their-four-wives-survived-the-Flood-How-many-people-could-a-family-of-8-procreate-in-say-500-years

[2] https://www.theatlantic.com/technology/archive/2024/01/quora-tragedy-answer-websites/677062/

[3] https://en.wikipedia.org/wiki/Spherical_cow

number of children who survive each generation.

Table 2 Population After 500 Years

Children m per Generation	log m	20 log m - 6	Population after 500 years	
			$8\times10^{20\log m - 6}$	$8(m/2)^{20}$
2	0.30103	0.02	$= 8\times10^{0.02} \approx 8\times1 = 8$ (steady state)	8
4	0.60206	6.04	$\approx 8\times10^{6} = 8$ million	$8\times1.05\times10^{6} \approx 8$ million
6	0.77815	9.56	$= 8\times10^{9.56} = 8\times3.6\times10^{9} \approx 29$ billion	$8\times3.5\times10^{9} = 28$ billion
8	0.90309	12.06	$\approx 8\times10^{12} = 8$ trillion = 8,000 billion	$8\times1.1\times10^{12} \approx 9$ trillion

Since the current population of the world is approximately 8 billion,[4] it might be interesting to know how many surviving children per generation would it take for Noah and his family to produce the current world's population after 500 years. So we want to know for what m would we have

$$8\times10^{20\log m - 6} = 8\times10^{9}$$

Then we want $$20 \log m - 6 = 9$$

or $$\log m = ¾ = .75$$

or $$m = 10^{.75} \approx 5.62$$

This is consistent with Table 2, since it falls between 4 and 6 children per generation.

Population Growth Rate

Recall that the growth rate of a population P is defined as the constant r, where $dP/dt = r\,P$ so

$$r = dP/dt\ /\ P \text{ (change in population per year per total population)}$$

The rate r is usually given as a percentage ($100r$ %). Recall also that the solution to the differential equation defining r is

$$P = P_0e^{rt}$$

were P_0 is the initial population and P is the population at time t. So if P is the population after one generation of 25 years, then

$$r = {}^1/_{25} \ln P/P_0$$

Using the population approximation from Table 1, we have

$$r = {}^1/_{25} \ln (m/2) = 0.04 \ln (m/2)$$

So when the number of children m each generation is 2, we have $r = 0.04 \ln 1 = 0$. And so $P = P_0$ for all time, that is, the population remains constant as we saw in Table 2. Table 3 shows the growth rates for the other cases of the number m of children per generation.

Table 3 Growth Rates from Population Approximation

Children m per Generation	Annual Population Growth Rate	
	$(r = 0.04 \ln (m/2))$	%
2	$r = 0.04 \ln 1 = 0$	0%
4	$r = 0.04 \ln 2 = 0.04 \times 0.6931 = 0.0277$	2.8%
6	$r = 0.04 \ln 3 = 0.04 \times 1.0986 = 0.0439$	4.4%
8	$r = 0.04 \ln 4 = 0.04 \times 1.3863 = 0.0555$	5.6%

[4] https://www.census.gov/library/stories/2023/11/world-population-estimated-eight-billion.html

When we consider populations as a whole, our crude assumptions have not really taken into account parents who are still alive, people who did not have children, death rates, the availability of mates (and avoidance of incest), and the fact that births are spread out over time. The actual rate is certainly much lower.

Historically, apparently the population growth rate has been estimated at 0.04% annually over the period 10,000 BC to 1700 AD ([1], see also [2]). (To go from 4 million in 10,000 BC to 190 million in year 0 would actually mean a growth rate of .0386%.) This would mean after 500 years an initial population of 8 would grow to

$$P = 8\, e^{0.000386 \times 500} = 8 \times 1.2129 = 9.70 \text{ people},$$

virtually steady state. Even at 0.04% this growth rate is 100 times smaller that the 4.4% growth rate for 6 children per couple per generation of our crude approximation.

The Time of the Flood

It doesn't really matter when the supposed Biblical flood occurred, which would be the time of Noah and his sons, since the whole period had a population growth rate of 0.0386%. According to Biblical calculations the period might be around 5,000 BC, or from some possible geological events around 12,000 to 9,000 BC ([3]). So these times do not offer any improvement over the low growth.

It looks like in 10,000 BC there were around 1 million people and from 8,000 BC to 5,000 BC 5 million people. These numbers lead to the world population in 1700 AD of around 600 million. ([2])

How much might we infer that the Noah contingent contributed? If we start at 8,000 BC, this would be a period of 9,700 years to 1700 AD. So we would have at the 0.0386% growth rate,

$$P = 8\, e^{0.000386 \times 9700} = 8 \times 42.2752 = 338.20 \text{ people},$$

a negligible contribution out of 600 million.

Actually there is a problem. Going from 5 million to 600 million in 9700 years yields a growth rate of

$$r = {}^{1}/_{9700} \ln 600/5 = 4.7875 / 9700 = 0.000494 \text{ or } 0.0494\% \approx 0.05\%,$$

which is well above the 0.0386% rate. So if we use this new growth value for Noah, we get

$$P = 8\, e^{0.000494 \times 9700} = 8 \times 120.518 = 964.14 \text{ people},$$

which is a slight improvement. But still 1000/600,000,000 = 0.00000167 or a minuscule 0.000167% of the population in 1700AD.

References

[1] Roser, Max and Hannah Ritchie, "How Has World Population Growth Changed Over Time?", OurWorldInData.org, 1 June 2023. (https://ourworldindata.org/population-growth-over-time, retrieved 12/17/2023)

[2] U.S. Census, "Historical Estimates of World Population", 5 December 2022. (https://www.census.gov/data/tables/time-series/demo/international-programs/historical-est-worldpop.html, retrieved 12/17/2023)

[3] "Genesis flood narrative", *Wikipedia*, (https://en.wikipedia.org/wiki/Genesis_flood_narrative, retrieved 12/17/2023)