# More Right Triangle Magic

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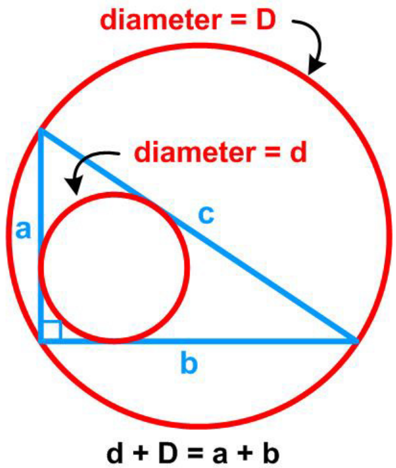


James Tanton asked to prove the following surprising property of a right triangle and its circumscribed and inscribed circles.[1]

Every triangle is circumscribed by some circle of diameter D, say, and circumscribes another circle of smaller diameter d. For a right triangle, d + D equals the sum of two side lengths of the triangle. Why?

## My Solution

Our approach is captured in Figure 1. Since the sides of the right triangle are tangent to the inscribed circle, they are perpendicular to the radii of the circle, as shown. Since the sides are at a right angle to each other, they form a d/2 x d/2 square with the radii. Since tangents to a circle from a common point are the same length, we have

$$c = (a - d/2) + (b - d/2)$$

so

$$c + d = a + b.$$

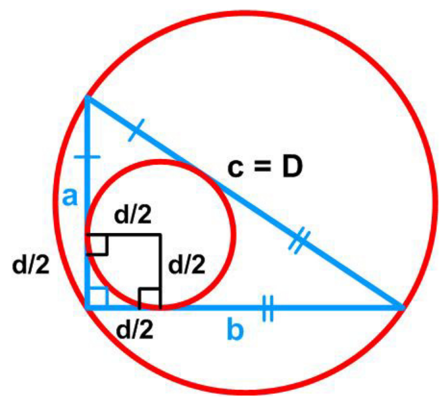


**Figure 1**

But the hypotenuse c is also a diameter of the circle. (The central angle for the inscribed 90° angle is twice its value, or 180°, making a straight line through the center, namely, the diameter of the circle.) Therefore c = D and so

$$D + d = a + b.$$

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[1] https://twitter.com/jamestanton/status/1536299591183867906. 13 June 2022.