Two Containers Mixing Puzzle

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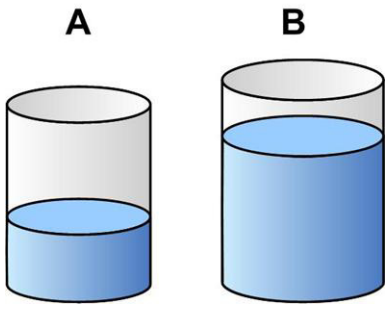

This is a slightly different type of a mixture problem from Dan Griller ([1]).

Two containers A and B sit on a table, partially filled with water. First, 40% of the water in A is poured into B, which completely fills it. Then 75% of the water in B is poured into A, which completely fills it. 80% of the water in A is poured into B, which completely fills it. Calculate the ratio of the capacity of container A to the capacity of container B, and the fraction of container A that was occupied by water at the start.

Solution

Again, the key to solving mixture problems like this is to convert from percentages to actual amounts. Let A be the capacity of container A and B the capacity of container B. Further let a be the amount of water in container A and b the amount of water in container B (Figure 1).

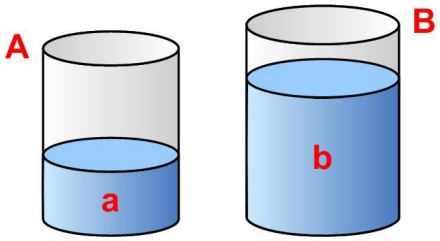

Figure 1

Then the three steps in the problem yield the following three equations

$$B = .4a + b \tag{1}$$

$$A = .75B + .6a \tag{2}$$

$$B = .8A + .25B \tag{3}$$

From equation (3) we immediately get the ratio of A to B, namely,

$$A/B = (^3/_4)(^{10}/_8) = 15/16.$$

Using this result with equation (2) yields

$$A = (^3/_4)(^{16}/_{15}\, A) + .6a$$

or

$$.2 = .6\ a/A$$

or

$$a/A = {}^1/_3.$$

References

[1] Griller, Daniel, *A Ring of Cats and Dogs and Other Curious Puzzles*, Rational Falcon, 2022. Diamond Problem #21. (Scale of difficulty: Bronze, Silver, Gold, Diamond.)