Two More Jugs

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Here is another classic example of the three jug problem posed in the *Mathigon* Puzzle Calendars for 2017 ([1]).

How can I measure exactly 8 liters of water, using just one 11 liter and on 6 liter bucket?

It is assumed you have unlimited access to water (the "third jug" of at least 17 liters). You can only fill or empty the jugs, unless in poring from one jug into another you fill the receiving jug before emptying the poring jug. (Hint: see the Three Jugs Problem.[1])

Solution

As discussed in the Three Jugs Problem, the puzzle is modeled as a skew billiard table with a grid of equilateral triangles where each vertex represents a state of the water in the jugs (Figure 1). For example, a point (4, 7) represents 4 liters of water in the 6 liter jug and 7 liters of water in the 11 liter jug. However, the only states allowed via emptying and filling the jugs lie on the edges of the grid. (4, 7) is an interior point and so cannot be obtained from legal moves in the problem. The state we want is (0, 8) where the 11 liter jug contains 8 liters and the 6 liter jug is empty.

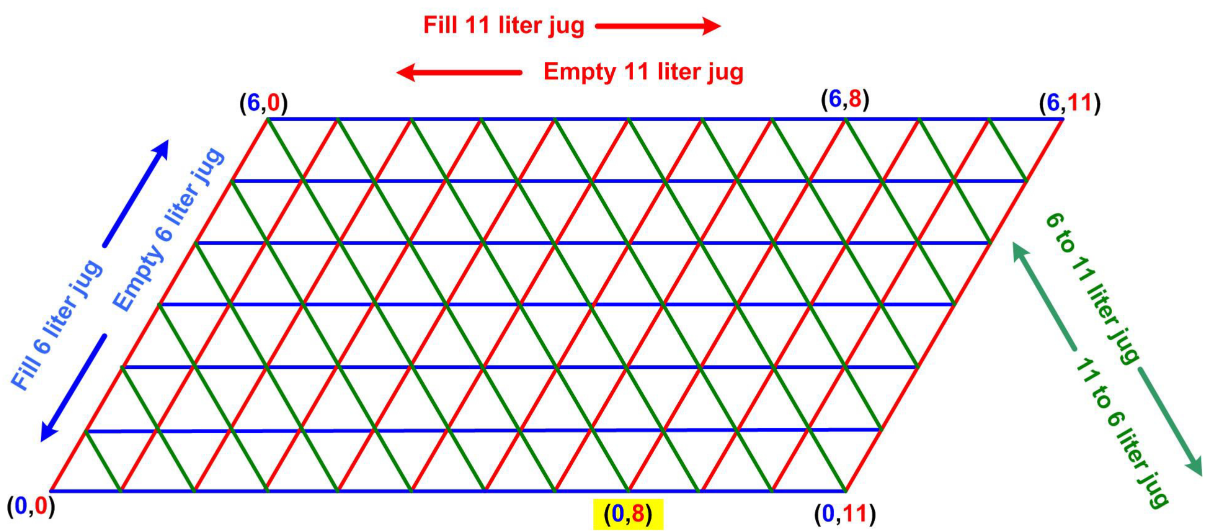
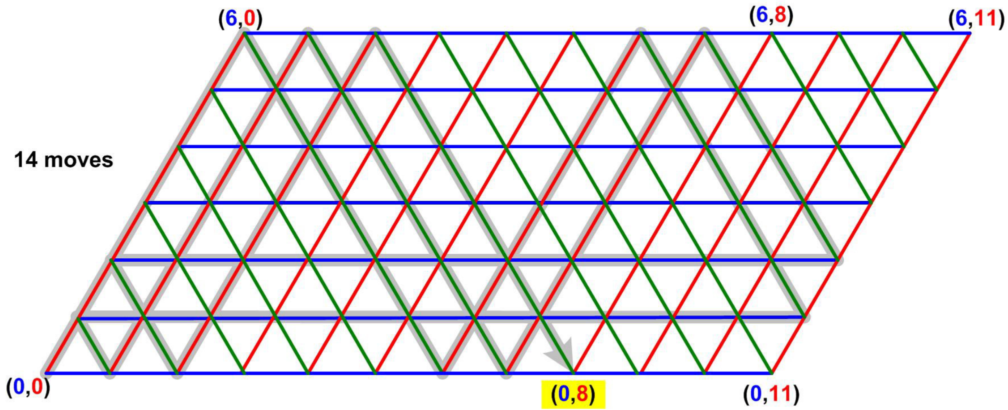


Figure 1 Problem Setup (skew billiard table model)

A "move" in the solution is to run along a line from one edge of the grid to the other. Successive moves are paths a billiard ball would follow if it bounced off the edge at the same angle it struck the edge, namely 60°. Therefore all moves are along the colored lines shown in the grid. A move along a red line leaves the contents of the 11 liter jug unchanged and either fills or empties the 6 liter jug. Similarly, a move along a blue line leaves the contents of the 6 liter jug unchanged and either fills or empties the 11 liter jug. A move a long a green line pours water from the 6 liter jug into the 11 liter jug or vice versa.

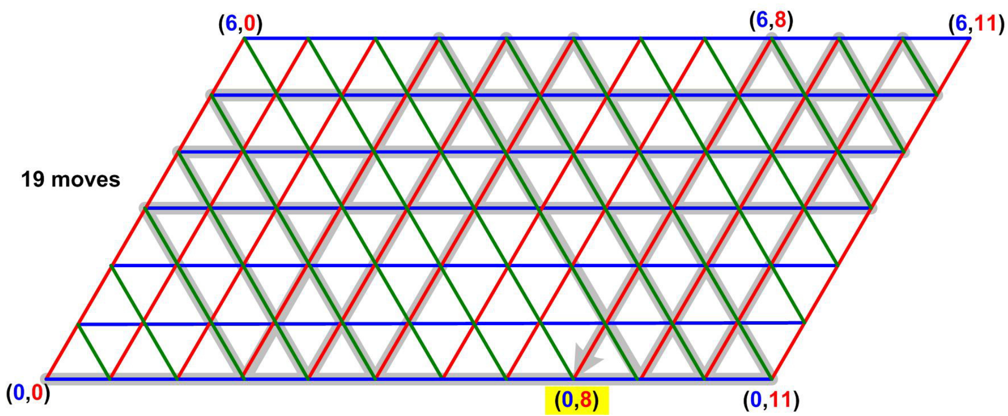
[1] https://josmfs.net/2018/12/29/three-jugs-problem/

The solutions are given in Figure 2 (filling the 6 liter jug first) and Figure 3 (filling the 11 liter jug first).



Pouring Sequence	
6 liter	11 liter
0	0
6	0
0	6
6	6
1	11
1	0
0	1
6	1
0	7
6	7
2	11
2	0
0	2
6	2
0	8

Figure 2 14 Move Solution



Pouring Sequence	
6 liter	11 liter
0	0
0	11
6	5
0	5
5	0
5	11
6	10
0	10
6	4
0	4
4	0
4	11
6	9
0	9
6	3
0	3
3	0
3	11
6	8
0	8

Figure 3 19 Move Solution

References

[1] "Number 17, Puzzle Calendars 2017", *Mathigon* (https://mathigon.org/puzzles#2017)