

Al the Chemist I

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This is a relatively simple problem from the inventive Raymond Smullyan in the “Brain Bogglers” section of the 1996 *Discover* magazine ([1]).

AL THE CHEMIST—not an alchemist, though his name might suggest it—one day partially filled a container with some concoction or other. He knew the volume of fluid in the container, as well as the volume of empty space, and realized that two-thirds of the former was equal to four-fifths of the latter. Was the container then less than half full, more than half full, or exactly half full?

Solution

Let V be the volume of the entire container and x the volume of the fluid in the container. Then the statement of the problem says

$$\frac{2}{3}x = \frac{4}{5}(V - x)$$

or

$$\frac{2}{3}x/V = \frac{4}{5}(1 - x/V)$$

So

$$\left(\frac{22}{15}\right)x/V = \frac{4}{5}$$

or

$$x/V = \frac{6}{11} > \frac{1}{2}$$

So the container is more than half full.

References

[1] Smullyan, Raymond, “Brain Bogglers: Al the Chemist”, *Discover*, August 1996

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