Right Angles in a Square

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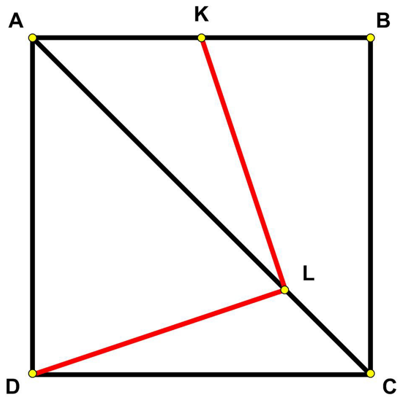

Here is another problem from the "Challenges" section of the *Quantum* magazine ([1]).

Point L divides the diagonal AC of a square ABCD in the ratio 3:1, K is the midpoint of side AB. Prove that angle KLD is a right angle. (Y. Bogaturov)

My Solution

I am afraid I proceeded in a rather pedestrian manor, relying more on analytic geometry than plane geometry (Figure 1). I assumed the square was a unit square.

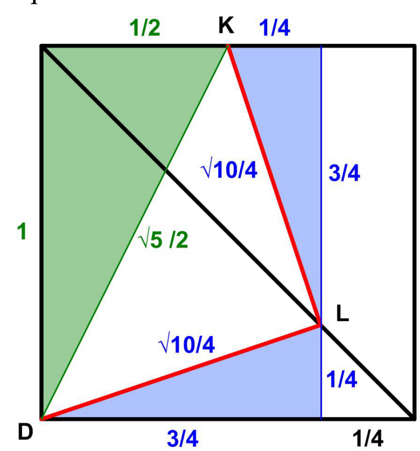

Figure 1 My Solution

Drawing a line from D to K forms the green right triangle in the figure. Since K is the midpoint of the side of the square, the legs of the right triangle are ½ and 1. Thus the hypotenuse is $\sqrt{5}/2$.

Drawing a second, vertical line through L forms two blue right triangles in the figure. Since L is ¼ of the distance from the lower right vertex, the vertical line forms an isosceles right triangle with legs ¼. Therefore, the horizontal leg of the upper blue triangle is also ¼, and the blue triangles are congruent with legs ¼ and ¾. Therefore the equal hypotenuses are $\sqrt{10}/4$. Since

$$(\sqrt{10}/4)^2 + (\sqrt{10}/4)^2 = 2\cdot 10/16 = 5/4 = (\sqrt{5}/2)^2,$$

by the converse of the Pythagorean Theorem triangle KLD is a right triangle and KLD is a right angle.

Quantum Solution

The *Quantum* solution is pure plane geometry and quite elegant.

Divide the given square into $4\cdot 4 = 16$ congruent squares (Figure 2). Then points L and K turn out to be nodes of the grid obtained. The rotation of the grid through 90º about point L obviously takes triangle LMD into triangle LNK. So angle DLK is equal to the rotation angle—that is, to 90º (and, in addition, LD = LK).

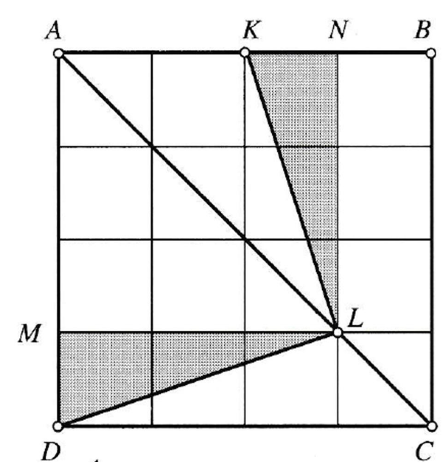

Figure 2 *Quantum* Solution

References

[1] "Challenges" *Quantum Magazine*, Vol. 2 No. 2, National Science Teachers Assoc., Springer-Verlag, Nov-Dec 1991 p.18 M36