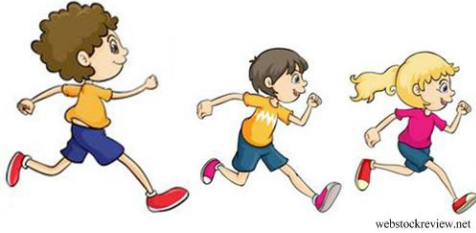


Handicap Racing

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This is a nice variation on a racing problem by Geoffrey Mott-Smith from 1954 ([1]).

On one side of the playground some of the children were holding foot-races, under a supervisor who handicapped each child according to age and size. In one race, she placed the big boy at the starting line, the little boy a few paces in front of the line, and she gave the little girl twice as much headstart over the little boy as he had over the big boy. The big boy won the race nevertheless. He overtook the little boy in 6 seconds, and the little girl 4 seconds later.

Assuming that all three runners maintained a uniform speed, how long did it take the little boy to overtake the little girl?

My Solution

Let v_B , v_b , v_g be the speeds of the big boy, little boy, and little girl respectively. And let D be the headstart the little boy has over the big boy. Then $2D$ is the headstart the little girl has over the little boy. The information in the problem is shown in the space-time diagram in Figure 1, where the big boy passes the little boy in 6 seconds and passes the little girl in 10 seconds, and where the little boy passes the little girl in T seconds.

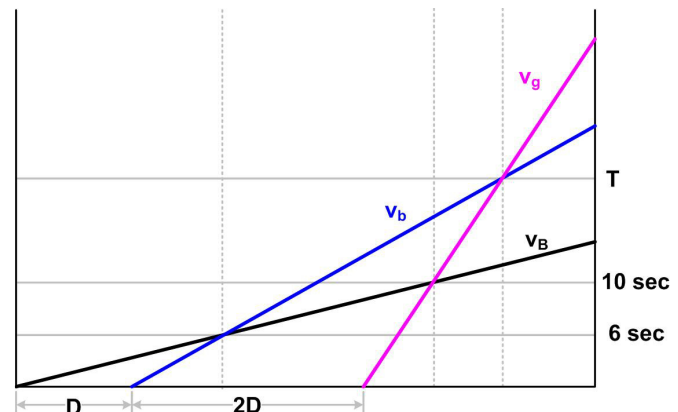


Figure 1

Then we have the following numerical relations for distances:

$$v_B \cdot 6 = v_b \cdot 6 + D$$

$$v_B \cdot 10 = v_g \cdot 10 + 3D$$

$$v_b \cdot T = v_g \cdot T + 2D$$

Combining the first two equations we get

$$v_b + \frac{1}{6} D = v_g \cdot T + \frac{3}{10} D$$

or

$$v_b - v_g = \frac{2}{15} D.$$

From the third equation we get

$$(v_b - v_g) T = 2D.$$

So

$$(v_b - v_g) T = 15 (v_b - v_g),$$

or

$$T = 15 \text{ seconds}$$

Mott-Smith Solution

Actually, his solution is just the answer.

The little boy overtook the little girl 15 seconds after the start of the race, or 5 seconds after the big boy passed her.

References

- [1] Mott-Smith, Geoffrey, "63. Handicap Racing," *Mathematical Puzzles for Beginners & Enthusiasts*, Blakiston Co, 1946, 2nd revised edition, Dover Publications, 1954.