Ant Connection Problem

30 January 2022

Jim Stevenson

This is a nice puzzle from the Maths Masters team, Burkard Polster (aka Mathologer) and Marty Ross ([1]) as part of their "Summer Quizzes" offerings.

Two ants are on a cylindrical glass that is 5 centimetres in diameter. The ants are on opposite sides of the glass, 5 centimetres down from the glass's rim. If both ants are on the outside of the glass, what is the shortest distance required for one ant to crawl to the other? What if one ant is on the outside of the glass and the other is on the inside?

Solution

If we treat the glass as a cylinder open at both ends, we can slice it open with a vertical cut and unroll it onto a flat plane (Figure 1). Then the distance between the ants is half of the circumference of the glass or ½ 5π, or approximately 7.9 cm.

For the case where the blue ant is inside the glass, we could imagine the glass to be double-walled and hinge the inside wall up along the rim of the glass (Figure 2). Now the position of the blue ant is as shown in Figure 3 and the shortest path between the red and blue ants is along the hypotenuse of a right triangle with sides $^5/_2\pi$ and 10 cm. Thus the distance between the ants is

$$5\sqrt{\left(\frac{\pi}{2}\right)^2 + 2^2}$$

or approximately 12.7 cm.

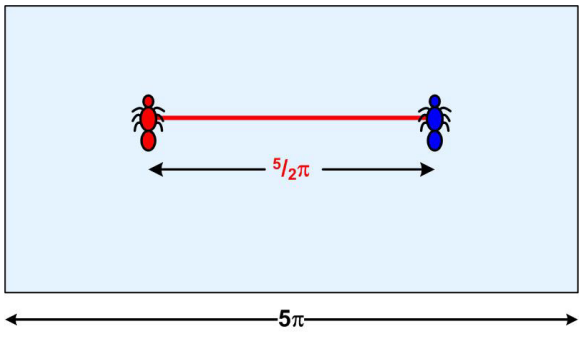

Figure 1

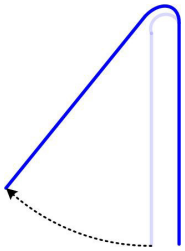

Figure 2

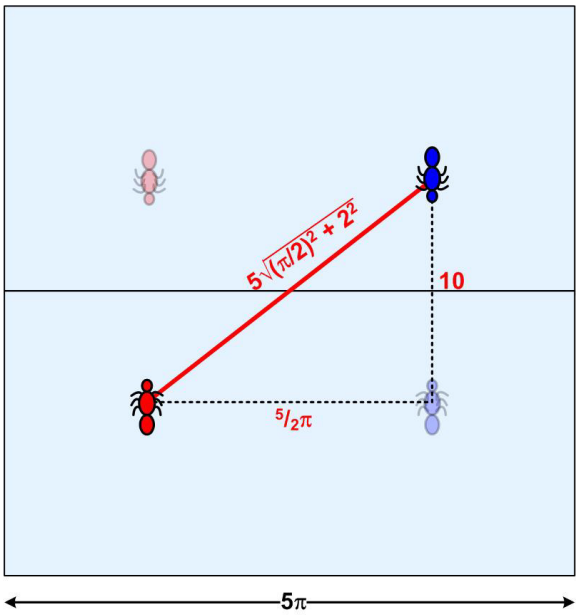

Figure 3

The Maths Masters' solution is the same.

References

[1] Polster, Burkard and Marty Ross, "The Maths Masters' Summer Quiz, Problem Hard 2", *The Age*, 21 November 2011 (https://www.qedcat.com/summerquizzes/2010%20QUIZ.pdf)