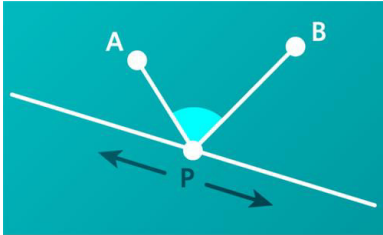


Max Angle Puzzle

16 January 2023

Jim Stevenson



Here is a familiar puzzle from the *Mathigon* Puzzle Calendars for 2021 ([1]).

Given a line and two points A and B, which point P on the line forms the largest angle APB?

Solution

This is really a special case of the “Regiomontanus 1471 Problem.”¹

The idea is to pass a circle through the two points and intersect the line. These intersection points are candidates for the point P. Notice that the inscribed angles in the circle from these intersection points are both equal to each other. We look at different circles intersecting the line. All the centers lie on the perpendicular bisector of the line between the two initial points. As the candidate circles move up the perpendicular bisector, the equal inscribed angles get larger until the circle just touches the line tangentially and the intersection points converge to one point P. That tangent point P will give the largest inscribed angle.

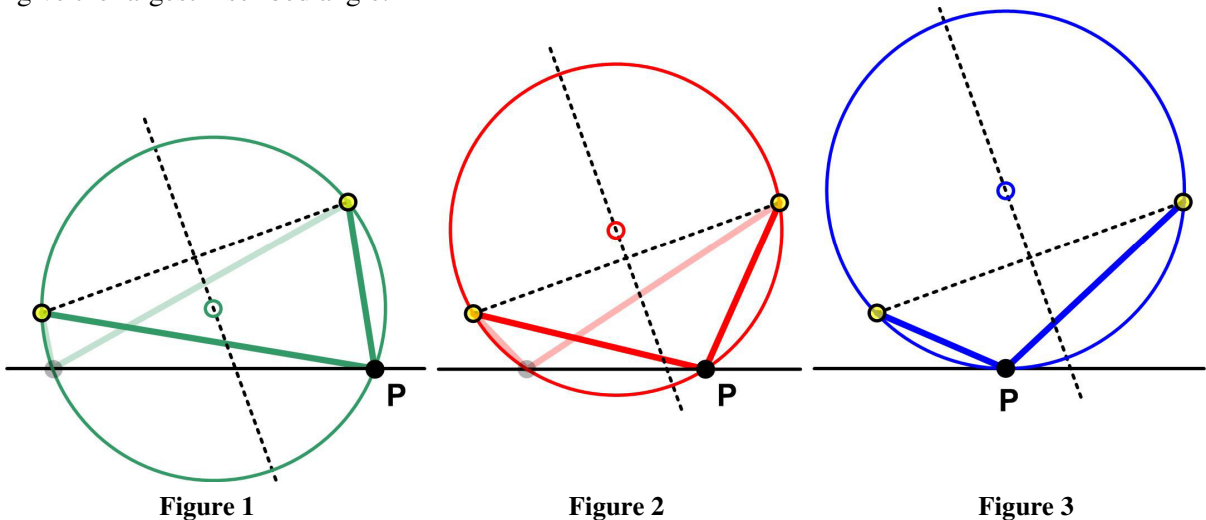


Figure 1

Figure 2

Figure 3

Comment. A great application of this idea is given in a rugby example by Numberphile ([2]), which reveals even more fascinating properties of the solution.

References

[1] “Number 10, Puzzle Calendars 2021”, *Mathigon* (<https://mathigon.org/puzzles#2021>)

[2] “The Perfect Goal Kicking Angle” *Numberphile* 16 March 2023. (<https://www.youtube.com/watch?v=rHdYv62F5fs>)

© 2023 James Stevenson

¹ <http://josmfs.net/2019/03/24/regiomontanus-1471-problem/>