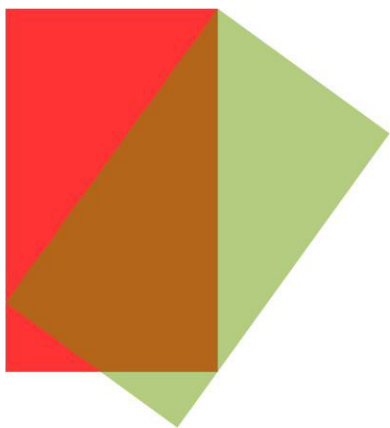


Covering Rectangle Puzzle

30 January 2022

Jim Stevenson

This is a nice puzzle from the Maths Masters team, Burkard Polster (aka Mathologer) and Marty Ross ([1]) as part of their “Summer Quizzes” offerings.



In the picture does the green rectangle cover more or less than half of the [congruent] red rectangle?

My Solution

It became clear that the problem assumed the two rectangles were the same, so I made it explicit.

A sequence of arguments shows the red and green pairs of triangles in Figure 1 are congruent. In particular, the upper edges of the large triangles are equal, being from the same rectangles, and the pivoting of the green rectangle over the red one means the two angles at the pivot point are equal. Since they are right triangles, they are similar, and with a common side, they are therefore congruent. Furthermore, the smaller pair of red and green right triangles are similar and with a common sized side they must be congruent.

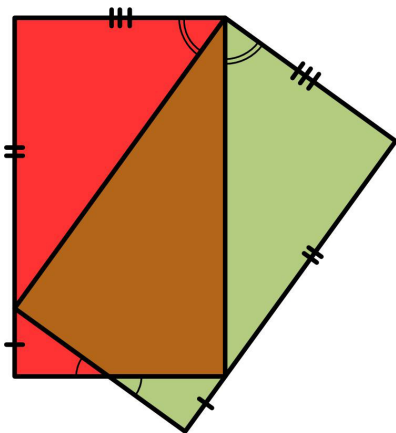


Figure 1

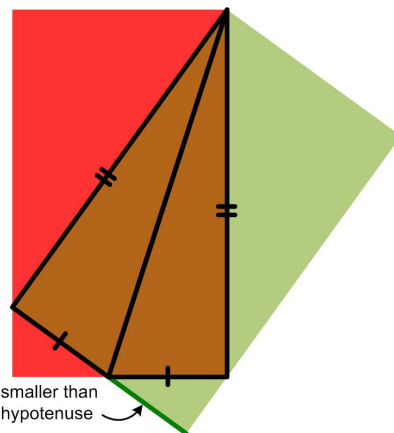


Figure 2

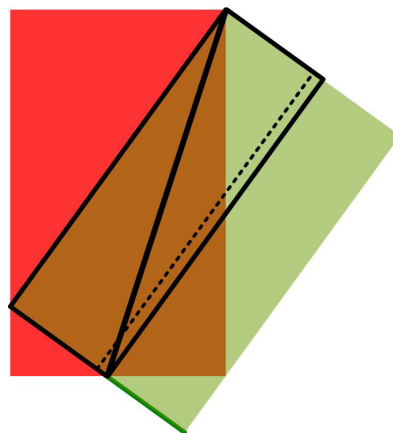


Figure 3

From the congruencies of the red and green pairs of triangles we get the congruence of the two right triangles in the overlay (Figure 2). Notice that this means the green line on the small green triangle is shorter than the hypotenuse or leg of one of the covering triangles. That means the base of one of the covering triangles is larger than half the edge of the rectangle. This in turn implies the covering area is larger than half the area of the rectangle (Figure 3).

Maths Masters Solution

The Maths Masters’ solution is much slicker.

Answer: More than half.

Solution: The blue triangle pictured below (Figure 4) is exactly half of the red rectangle.

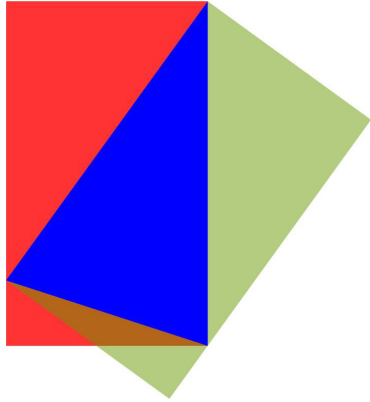


Figure 4

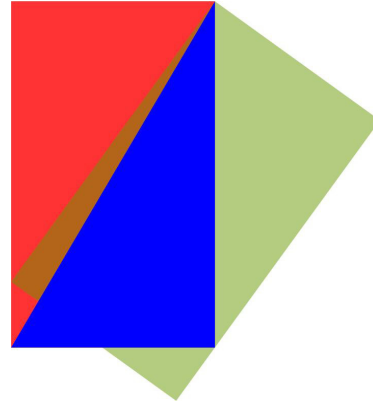


Figure 5

This can be seen via Figure 5 where the blue triangle has been sheared to make it clear that its area is half the rectangle's

References

- [1] Polster, Burkard and Marty Ross, "The Maths Masters' Summer Quiz, Problem Medium 5", *The Age*, 9 December 2013 (<https://www.qedcat.com/summerquizzes/2013%20QUIZ.pdf>)

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