Refabulating Widgets

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This is a work problem from Geoffrey Mott-Smith from 1954 ([1]).

"If a man can do a job in one day, how long will it take two men to do the job?"

No book of puzzles, I take it, is complete without such a question. I will not blame the reader in the least if he hastily turns the page, for I, too, was annoyed by "If a man" conundrums in my schooldays. Besides, the answer in the back of the book was always wrong. Everybody knows it will take the two men two days to do the job, because they will talk about women and the weather, they will argue about how the job is to be done, they will negotiate as to which is to do it. In schoolbooks the masons and bricklayers are not men, they are robots.

Strictly on the understanding that I am really talking about robots, I will put it to you:

If a tinker and his helper can refabulate a widget in 2 days, and if the tinker working with the apprentice instead would take 3 days, while the helper and the apprentice would take 6 days to do the job, how long would it take each working alone to refabulate the widget?

My Solution

This problem has a surprise solution. Let w be the number of widgets made, d the number of days, and r_T, r_H, and r_A the rates of widget refabulation by the tinker, helper, and apprentice, respectively. The general model is $w = r \cdot d$ for the widget refabulation. The problem says

$$1 = (r_T + r_H)\cdot 2 = 2r_T + 2r_H$$

$$1 = (r_T + r_A)\cdot 3 = 3r_T + 3r_A$$

$$1 = (r_H + r_A)\cdot 6 = 6r_H + 6r_A$$

Multiply the first equation by 3 and the second by 2 so that

$$3 = 6r_T + 6r_H$$

$$2 = 6r_T + 6r_A$$

$$1 = 6r_H + 6r_A$$

Subtract the second equation from the first to get

$$1 = 6r_H - 6r_A$$

$$1 = 6r_H + 6r_A$$

Adding these two equations yields

$$2 = 12r_H \quad \Rightarrow \quad r_H = 1/6$$

Subtracting the first from the second yields

$$0 = 12r_A \quad \Rightarrow \quad r_A = 0$$

The apprentice cannot refabulate any widgets on his own! Plugging this value into the second of the original equations yields

$$r_T = 1/3$$

So the tinker takes 3 days to refabulate a widget, the helper 6 days, and the apprentice never.

Mott-Smith Solution

Again, Mott-Smith just provides the answer.

Working alone, the tinker would take 3 days and the helper would take 6. The apprentice would never finish the job at all—in fact, he turned out to be entirely useless.

References

[1] Mott-Smith, Geoffrey, “#68 If a Man Can Do a Job”, *Mathematical Puzzles for Beginners & Enthusiasts*, Blakiston Co, 1946, 2nd revised edition, Dover Publications, 1954. p.34.