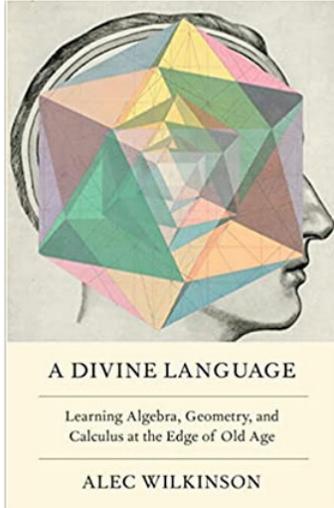


A Divine Language

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Jim Stevenson



I have just finished reading a most remarkable book by Alec Wilkinson, called *A Divine Language: Learning Algebra, Geometry, and Calculus at the Edge of Old Age* ([1]). I had read an essay of his in the *New Yorker* ([2]) that turned out to be essentially excerpts from the book. I was so impressed with his descriptions of mathematics and intrigued by the premise of a mature adult in his 60s revisiting the nightmare of his high school experience with mathematics that I was eager to see if the book was as good as the essay. It was, and more.

The book is difficult to categorize—it is not primarily a history of mathematics, as suggested by Amazon. But it is fascinating on several levels. There is the issue of a mature perspective revisiting a period of one’s youth; the challenges of teaching a novice mathematics, especially a novice who has a strong antagonism for the subject; and insights into why someone would want to learn a subject that can be of no “use” to them in life, especially their later years.

Wilkinson has a strong philosophical urge; he wanted to understand the role of mathematics in human knowledge and the perspective it brought to life. He was constantly asking the big questions: is mathematics discovered or invented, what is the balance between nature and nurture, why does mathematics seem to describe the world so well, what is the link between memorization and understanding, how do you come to understand anything?

He clearly is an excellent writer, though I haven’t had the opportunity to read any of his other writing. His use of metaphor is outstanding. Even with his rudimentary understanding of mathematics he managed to capture many facets of its essence and then express them vividly. From the essay (and the book):

What did I learn? ... That mathematics is both real and not real. Like novelists and musicians, mathematicians produce thought objects that have no presence in the physical world. ... The known parts are ephemeral, but also concrete for being true, and more reliable and everlasting than any object in the physical world. ...

An imaginary world’s being infallible is very strange. This spectral quality is bewildering, even to mathematicians. The mathematician John Conway once said, “It’s quite astonishing, and I still don’t understand it, despite having been a mathematician all my life. How can things be there without actually being there?” ...

Someone who says that human beings created the operations of arithmetic cannot say that we created the results. ...

Numbers did not initially provoke wonder or reverence. They did that later, partly because they invoked notions of infinity and therefore of God and, after that, because they appeared to be a language in which nature could be expressed. And because, on examination, numbers showed themselves to be complex in ways that had nothing to do with what we thought about them.([2])

Wilkinson spends a fair amount of time exploring what creators of mathematics do and what that is like—mathematical research, in other words. Why a novice in mathematics might do this, I believe, is attributable to the unique way he went about revisiting algebra and geometry, and learning calculus—he enlisted the aid of his niece, Amie Wilkinson, who was a research mathematician at the

University of Chicago. He learned from her and also her mathematician husband what it was like to do research in pure mathematics. But what was most amazing to me was that he employed Amie to guide his mathematical studies. She suggested he get a tutor at one point, but Alec demurred, since he felt that was like cheating and would not represent his own accomplishment. Perhaps you have to have gone through the four years of high school math, four years of college math, and four to six years of graduate math plus a decade or so of research to appreciate the distance between Amie's capabilities and mind-set and Alec's, where at one point at the end of their sojourn Amie admitted she was surprised at how little *arithmetic* Alec knew. Her saintly patience with his contentious labors was incredible. What we do for our relatives.

But there is a bigger point here I wanted to make. I believe Alec's struggles with learning math and his articulate description of his impediments and misunderstandings demonstrated clearly the value of good public school math teachers. They have an extensive experience with the difficulties young students face trying to learn math for the first time. They have a wealth of techniques and tools that can be adapted to each situation, if time allows, that a research mathematician would be oblivious to, or have long forgotten. Alec refers at one point to this distance and the feeling that Amie cannot fathom his misunderstandings.

The days of difficulty accrue. As much as Amie tries to help, the problem of someone's explaining something to you, especially something complex, is that they don't know what you don't know, or why it is hard for you to know it. To be taught a difficult subject intimately is sometimes to wonder how human beings manage to talk to each other at all. ([1] p.239)

And then there is this arresting metaphor about his inability to achieve comprehension:

It has been as if I were trying to read but because of some deficiency or inhibition saw only single words without understanding that they formed sentences. ([1] p.9)

Or this one:

Math and language may be connected in the minds of some linguists, but learning math does not seem to me to be like learning another language. It is like learning a language when you don't have one yet. ([1] p.227)

Besides the absence of a teacher perhaps better matched to his situation, there was another aspect that was missing from Alec's narrative—music. The biographical blurbs about him mention he was a rock and roll musician. I can't speak to the "musical" knowledge associated with a percussionist, but if he played a guitar or keyboard, he would experience what all of us who performed music have.

There are three levels of participation in music: listening, performing, and composing. I find they correspond roughly to levels of mathematical endeavors quite well. The average person with a public school math education might be at the level of a listener of music. One who pursues mathematics at the college level, or even the graduate level, is comparable to a performer. But a research mathematician is very like a composer. In research one has to create their own mathematics, often based on the efforts other mathematicians. But so must composers create their own music. It is a rarified activity, however, and only the few have the ability or talent to do it.

One important aspect of learning to perform music, however, is practice. From practice one can master the mechanics of their instrument so that they can really enjoy the music it makes, both alone and in concert with others. Alec seemed often to want to "understand" in depth each step he was trying to learn, which usually devolved into a lengthy interrogation and analysis. Amie would often say he was overthinking the situation and why didn't he just move on. In other words, go through the procedures until you achieved a level of unconscious mastery. Then things would start to "click". (There is a mystery there—how do we leap from an understanding of letters to that of words and then to sentences? It is more than just repetition. Still, see the articles by Barbara Oakley ([3]) and Tim

Gowers ([4]) on the importance of “fluency” in learning math.)¹

There were other aspects of Wilkinson’s story that I found arresting—for example, *Amie’s* challenge of the worth of word problems. Certainly Alec had nightmarish memories of word problems in his original encounters with algebra. “I recalled them as devious and irrational, the evil end of the language ladder whose higher end was poetry.”([1] p.62) But Alec actually had a rebuttal to Amie’s criticism.

Amie did not endorse my studying word problems. She said that they taught nothing of any lasting use and, once disposed of, never appeared in mathematics again, an argument I wish I had been equipped to present to Mr. Carmine Biazzo. I understood that they were a byway to her, but in the math curriculum, beginner division, they are prominent, at least as I remember it. Perhaps this is because ancient algebra begins with word problems. The earliest document with algebra on it is the Ahmes Papyrus, an Egyptian scroll in the British Museum that was copied by a scribe named Ahmes “in the year 33, in the 4th month of the inundation season.” Scholars believe that this might be around 1650 BCE. The scroll describes itself as “the entrance into the knowledge of all existing things and all obscure secrets, and it is thought to be a math textbook. In addition to tables concerning addition and multiplication, instruction on how to handle fractions, geometric definitions, and method for finding areas and volume, it has eighty-four problems, one of which, the twenty-fourth, an example of rhetorical algebra, begins, “A heap and its 1/7 part become 19. What is the heap?” ([1] p.64)

Naturally, I was somewhat taken aback by Amie’s dismissal. My entire website is mostly puzzles in the form of word problems. But when I viewed the situation from her point of view—doing research in pure mathematics, I understood. Alec noticed that many calculus problems were in the form of word problems as well. And when I think of problems in physics, chemistry, or biology, they also seem to be predominantly in the form of word problems. I believe the difference is captured in the modified Martin Gardner diagram I employed in my post “The Essence of Mathematics” (Figure 1).²

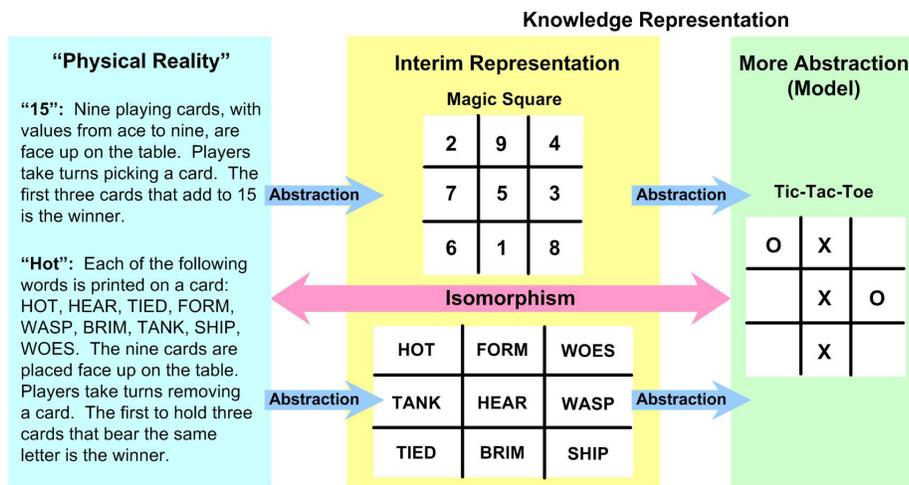


Figure 1 Martin Gardner Example of Mathematical Knowledge Representation

Research mathematicians explore the mathematics on its own, the abstract models in the diagram.

¹ Upon rereading a section of the book, I find I overlooked a moment when Alec did address a comparison with learning music, only he did not find the analogy that I did. “I studied music in college and sometimes when math was trampling on me now I would tell myself that it doesn’t make sense that someone can learn a complicated practice of one kind and not another, music or languages or philosophy, say, but not math.” ([1] p.10)

² <http://josmfs.net/2019/03/03/the-essence-of-mathematics/>

The analogy with tic-tac-toe would be to ask is there a winning strategy for a 4×4 grid? an $n \times n$ grid? a $4 \times 4 \times 4$ grid? etc., which goes beyond the original problem. In other words, the mathematician concentrates on the game itself and not the applications. It is the physicists, chemists, and biologist (and economists) who address “physical reality” and try to find mathematical models from the mathematician’s toolbox that they can apply to their problem (possibly with the aid of an applied mathematician who would come up with the mathematical equivalent of the magic square link). This “physical reality” is usually in the form of a giant, complex, ambiguous “word” problem.³

The importance of a mathematical relationship with “physical reality” as a motivator began to dawn on Alec as he grappled with calculus.

On a rough day with calculus, opening a textbook, I see a blizzard of numbers and symbols, and think of other things that I might rather be doing. Somewhat late in algebra, or maybe early in geometry, is when I realized that new knowledge lingers fitfully in the mind, and that it isn’t possible to remember everything I am learning. An acquaintance is what I am managing. What I wish is that in high school, while studying geometry, say, I had also been taught the ways in which it figures in painting and perspective, in architecture and in the natural world and so on, so that I could see it perhaps the way the Greeks had seen it. If I had felt that the world was connected in its parts, I might have been provoked to a kind of wonder and enthusiasm. I might have *wanted* to learn. In approaching mathematics now through its separate disciplines, I wonder if I have misaligned my purposes. Perhaps I should have done what Deane suggested and looked more for mathematics in the world and less on the page. ([1] pp.227-8)

As Alec describes his labors in calculus that seem to become mired in techniques for finding antiderivatives for solving integrals, I thought he might be better served if less time were spent on techniques he probably never will need and more on the power of calculus. One good example of this would be optimization problems that only require differentiation and setting derivatives to zero. Relating this to curves would marry calculus, algebra, and geometry in one application. And differential equations involving the exponential function would be a great springboard to problems of growth where the rate of growth of a quantity is proportional to the quantity remaining. In other words, downplay integration and concentrate on differentiation where more accessible and interesting examples can be found (though understanding how to go from gravitational acceleration, to velocity, to displacement is a basic and valuable example, and it does not require hard-to-find antiderivatives).

I have only tapped a small portion of this engaging book. There is so much more I could discuss. I imagine it would appeal to a wide audience because of its many levels and fluid writing. I am grateful that someone performed this unique endeavor and applaud his results. By the way, for all his profession of disappointment in his accomplishments, Alec Wilkinson managed to cover two years of high school algebra, one year of high school geometry, and one year of college level (though maybe now high school level) calculus, totaling over four years of effort in little over a year. That is no small achievement.

References

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- [3] Oakley, Barbara, “How I Rewired My Brain to Become Fluent in Math”, *Nautilus Magazine*,

³ For more of my defense of word problems, see my post “The Za’irajah and Mathematics” (<https://josmfs.net/2020/02/29/the-zairajah-and-mathematics/>)

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