Meeting on the Bridge

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Here is another Brainteaser from the *Quantum* math magazine ([1]).

Nick left Nicktown at 10:18 A.M. and arrived at Georgetown at 1:30 P.M., walking at a constant speed. On the same day, George left Georgetown at 9:00 A.M. and arrived at Nicktown at 11:40 A.M., walking at a constant speed along the same road. The road crosses a wide river. Nick and George arrived at the bridge simultaneously, each from his side of the river. Nick left the bridge 1 minute later than George. When did they arrive at the bridge?

My Solution

Figure 1 shows the setup. D represents the distance between Nicktown and Georgetown, D_1 the distance Nick travels from Nicktown to the bridge, D_2 the distance George travels from Georgetown to the bridge, and D_B the distance across the bridge. T is the time George takes to reach the bridge and T_B the time it takes for him to cross the bridge. Finally, v_N is Nick's walking speed and v_G is George's walking speed. We will represent time in minutes, so it takes Nick 3 hrs 12 min = 192 minutes to travel from Nicktown to Georgetown, and George 2 hrs 40 min = 160 minutes to travel from Georgetown to Nicktown. Nick starts 1 hr 18 min = 78 minutes after George has left.

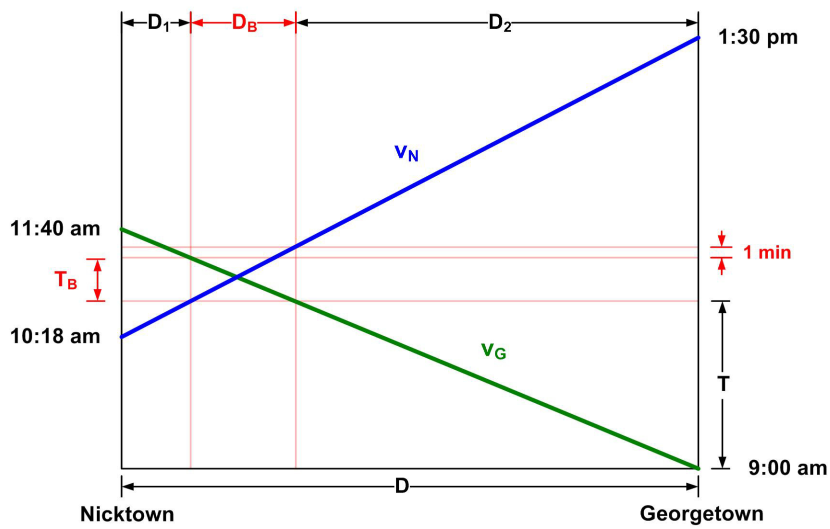

Figure 1 My Solution

So we have the following three equations.

(entire distance D) $$v_N\, 192 = D = v_G\, 160 \qquad (1)$$

(bridge length D_B) $$v_N\,(T_B + 1) = D_B = v_G\, T_B \qquad (2)$$

(bridge to Georgetown D_2) $$v_G\, T = D_2 = v_N\,(192 - (T - 78) - (T_B + 1)\,) \qquad (3)$$

Equation (1) implies

$$v_G / v_N = 6/5,$$

which together with equation (2) implies

$$T_B = 5 \text{ min.}$$

Substituting these values into equation (3) yields

$$(6/5)\ T = 264 - T$$

or
$$(11/5)\ T = 264$$

or
$$T = 120 \text{ min} = 2 \text{ hrs}$$

Therefore, Nick and George meet at the bridge at 9:00 + 2:00 = 11:00 AM.

Quantum Solution

It took Nick 3 hours 12 minutes—that is, 16/5 hours—to reach Georgetown, and it took George 2 hours 40 minutes—that is, 8/3 hours—to reach Nicktown. Denoting the distance between the towns by L miles, we find that Nick was walking at a speed of $5L/16$ mph and George's speed was $3L/8$ mph. We can determine the length of the bridge ℓ, since we know that George crossed it one minute faster than Nick:

$$16\ \ell/5L - 8\ \ell/3L = 1/60.$$

This yields $\ell = L/32$. Let t be the moment the boys reached the bridge. At this moment, the total distance walked by both boys was

$$L - L/32 = 31L/32.$$

On the other hand, this equals the sum of the distances walked by each of them—that is,

$$\frac{5L}{16}\left(t-\left(10+\frac{3}{10}\right)\right)+\frac{3L}{8}(t-9)$$

Setting these expressions equal to each other, we obtain

$$\frac{L}{16}\left(11t-\frac{211}{2}\right)=\frac{31L}{32}$$

which gives us t = 11 o'clock.

References

[1] "Meeting on the Bridge" B309 "Brainteasers" *Quantum* Vol.11, No.2, National Science Teachers Assoc., Springer-Verlag, Nov-Dec 2000. p.3