

Log Jam

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Here is a tricky little logarithm problem from the 2021 Math Calendar ([1]).

Find x , where

$$\log_2(\log_4(x)) = \log_4(\log_2(x))$$

As before, recall that all the answers are integer days of the month.

Solution

From the left-hand side we have $z = \log_4 x$ and $y = \log_4 z$ implies $4^z = x$ and $2^y = z$. And from the right-hand side we have $u = \log_4 x$ and $v = \log_4 u$ implies $2^u = x$ and $4^v = u$. So the equation says $y = v$. Therefore,

$$x = 4^{2^y} = 2^{2^{y+1}} \text{ and } x = 2^{4^v} = 2^{2^{2v}} = 2^{2^{2y}} \text{ since } y = v.$$

so

$$y + 1 = 2y$$

or

$$y = 1$$

Therefore,

$$x = 4^2 = 16.$$

References

- [1] Rapoport, Rebecca and Dean Chung, *Mathematics 2021: Your Daily epsilon of Math*, Rock Point, Quarto Publishing Group, New York, 2021. May

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