Two Candles

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This is another candle burning problem, presented by Presh Talwalkar.[1]

Two candles of equal heights but different thicknesses are lit. The first burns off in 8 hours and the second in 10 hours. How long after lighting, in hours, will the first candle be half the height of the second candle? The candles are lit simultaneously and each burns at a constant linear rate.

My Solution

Let L be the height of the both candles in inches, say. Let r_1 be the burn rate of the first candle, namely, $r_1 = L/8$ in/hr. Let r_2 be the burn rate of the second candle, namely, $r_2 = L/10$ in/hr. Let t be the time the first candle is half the height of the second. Then

$$L - r_1 t = (L - r_2 t)/2$$

or

$$L - (L/8)t = (L - (L/10)t)/2$$

or

$$2 - t/4 = 1 - t/10$$

So

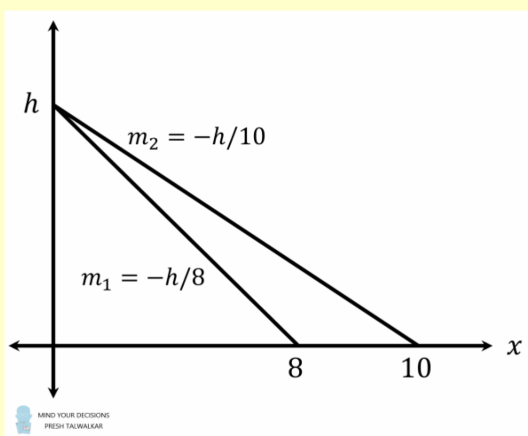
$$t(1/4 - 1/10) = 1$$

or

$$t = 20/3 \text{ hours} = 6 \text{ hr } 40 \text{ min}$$

Talwalkar's Solution

Consider graphing the height of each candle as time elapses. Both candles start at the same height *h* along the vertical axis. The first candle burns off in 8 hours while the second in 10 hours: these are the x-intercepts when the height is zero. The first candle's graph is between the points (0, *h*) and (8, 0) while the second's is between (0, *h*) and (10, 0).



The first candle's graph has a slope of (0 – *h*)/(8 – 0) = –*h*/8 and the second candle's graph has a slope of (0 – *h*)/(10 – 0) = –*h*/10. A general graph has an equation *y* = (slope)*x* + (y-intercept). Thus the two candles have graphs of:

[1] https://mindyourdecisions.com/blog/2022/03/22/two-candles-burning-half-the-height/

$$y_1 = (-h/8)x + h$$

$$y_2 = (-h/10)x + h$$

The first candle is half the height of the second is equivalent to the second candle being twice the height of the first. Thus we can solve:

$$y_2 = 2y_1$$

$$(-h/10)x + h = 2((-h/8)x + h)$$

$$-hx/10 + h = -hx/4 + 2h$$

$$h(-x/10 + 1) = h(-x/4 + 2)$$

$$-x/10 + 1 = -x/4 + 2$$

$$-x/10 + x/4 = 1$$

$$x(1/4 - 1/10) = 1$$

$$x(3/20) = 1$$

$$x = 20/3$$

Thus the answer is 20/3 hours, or 6 hours and 40 minutes.

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