Grandfather Clock Puzzle

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This is another doable puzzle from Sam Loyd ([1]).

BACK OF THE OLDTIME song of "Grandfather's clock was too tall for the shelf, so it stood for ninety years on the floor," there was a legend of a pestiferous grandfather and a cantankerous old clock which, from the fitful time when "it was bought on the morn, when the old man was born," it had made his whole life miserable, owing to an incurable habit which the clock had acquired of getting the hands tangled up whenever they attempted to pass.

These semi-occasional stoppages became of more frequent occurrence as advancing age made the old gentleman more irritable and his feeble hands more incapable of correcting the cranky antics of the balky old timepiece.

Once when the hands came together again and stopped the clock the old man flew into such an ungovernable passion that he fell down in a fit, stone dead, and it was then that

"The clock stopped short,
Never to go again,
When the old man died."

A photograph of the clock was presented to me, showing the classical figure of a female representing time, and it struck me as remarkable that with the knowledge of the hour and minute hands being together that it should be possible to figure out the exact time at which "the old man died," from the position of the second hand as shown, without having to see the face of the clock. The idea of being able to figure out the exact time of day from seeing the second hand alone is very odd, although not so difficult a puzzle as one would imagine.

My Solution

Again I followed the pattern developed for the "Fallen Clock Puzzle" posting.[1] and "A Question of Time" posting.[2] The key is to convert everything into minutes. From those posts:

> Suppose the time is given in h hours and m minutes. First, we wish to find out where the hour hand would be in minutes. Each hour represents 5 minutes on the clock and each minute is 1/60 of an hour and therefore 1/60 of a 5 minute interval. So in terms of minutes, the total time for the hour hand is given by
>
> $$h5 + (m/60)5 = 5h + m/12 \text{ (minutes)}$$
>
> The minute hand is given simply as m minutes.

[1] http://josmfs.net/2020/05/23/fallen-clock-puzzle/
[2] https://josmfs.net/2021/09/18/a-question-of-time/

In the current problem we will to find times when the two hands coincide. This will be when the hour hand (on the left) equals the minute hand (on the right):

$$5h + m/12 = m$$

or

$$m = h \cdot 60/11$$

Table 1 shows the values for the hours 1 to 11, where the decimals are rounded values. We see that when the hour and minute hands coincide at 9:49, they do so a little after 5 seconds past the minute, just as the sketch shows. No other hour comes close. So that is our answer: 9:49:05.45.

Table 1

hours	minutes	minutes	seconds
1	5.45	5	27.27
2	10.91	10	54.55
3	16.36	16	21.82
4	21.82	21	49.09
5	27.27	27	16.36
6	32.73	32	43.64
7	38.18	38	10.91
8	43.64	43	38.18
9	49.09	49	5.45
10	54.55	54	32.73
11	60.00	60	0.00

Loyd generally works in fractions, so for 9 hours we get

$$m = 540/11 = 49\ {}^{1}/_{11} \text{ minutes}$$

where

$${}^{1}/_{11} \cdot 60 = 5\ {}^{5}/_{11} \text{ seconds}$$

So the answer is 9 hours 49 minutes and $5\ {}^{5}/_{11}$ seconds.

Loyd Solution

Loyd's solution ([2]) concurs with mine.

"The clock stopped short,
Never to go again,
When the old man died,"

It was explained that the hour and minute hands had tangled up, and the puzzle was to determine their point of contact from the position of the second hand. Well, as the second hand may be said to be a little less than 5 ½ seconds past 60, we will find that the time must have been 49 minutes, 5 and ${}^{5}/_{11}$ths of a second past 9, which would bring the hour and minute hands together so that they caught and stopped the clock, which so irritated and excited grandfather that he just "up and died."

Of course, again Loyd doesn't explain how he got his answer.

References

[1] Loyd, Sam, "Puzzle of Grand Father's Clock," *Cyclopedia of Puzzles*, Lamb Publishing, New York, 1914. p.166

[2] ———, p.361