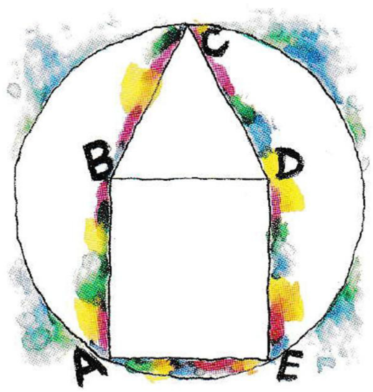
Circumscribed House Problem

23 September 2021

Jim Stevenson



Pavel Chernusky

Here is another problem from the “Brainteasers” section of the *Quantum* magazine ([1]).

Side AE of pentagon ABCDE equals its diagonal BD. All the other sides of this pentagon are equal to 1. What is the radius of the circle passing through points A, C, and E?

My Solution

Pass a circle of radius 1 centered at vertex C through the other vertices B and D (Figure 1). Then translate this circle 1 unit down until the original points at B and D coincide with points A and E (Figure 2). Then the top of the translated circle now passes through C and so must coincide with the original circle through A, C, and E. And so the radius of the original circle must be 1 (Figure 3).

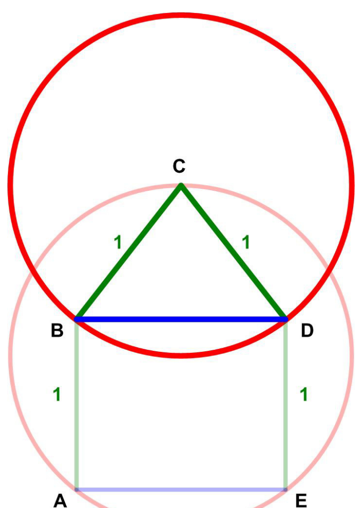


Figure 1

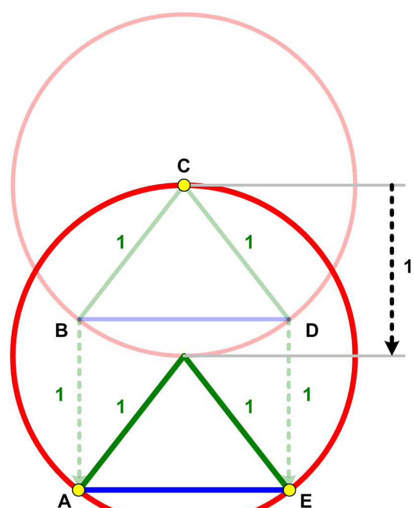


Figure 2

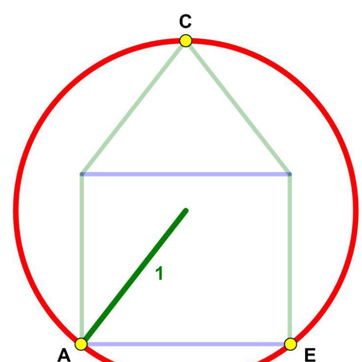


Figure 3

There probably should be some more explicit details in justifying the translation does what I claim, such as for example, quadrilateral ABDE is a rectangle and so the sides AB and ED are vertical and coincide with the translation path.

Quantum Solution

Let's construct a triangle *AOE* congruent to *BCD* as shown in Figure 4. It follows from the condition of the problem that *ABCO* and *EDCO* are rhombuses. Indeed, in quadrilateral *ABDE*, *BD* = *AE*, and *AB* = *DE*. Thus it is a parallelogram, so *AB* || *DE* and *AE* || *BD*. Now *CD* and *OE* make equal angles with the parallel lines *BD* and *AE*, and so *OE* || *CD*, so that *OEDC* is a rhombus. It's clear that *O* is the center of the desired circle, and its radius equals 1.

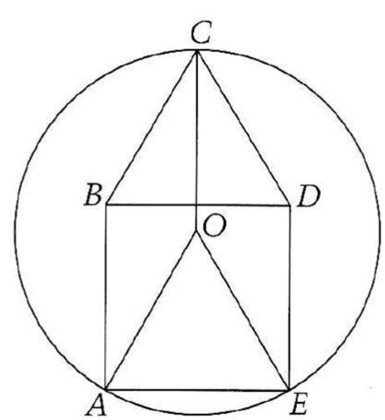
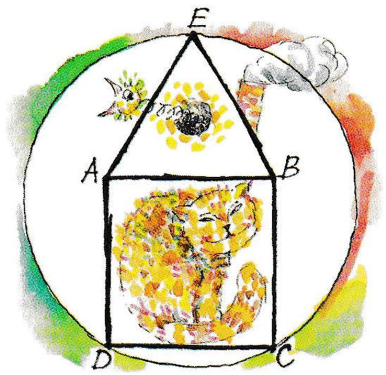


Figure 4 ***Quantum*** **Solution**



Pavel Chernusky

It turns out the *Quantum Magazine* had an earlier problem that was a special case of this one, where all the sides of the quadrilateral are equal ([2]).

An equilateral triangle *ABE* is constructed on the top of a square *ABCD* (see the figure). Find the radius of the circle drawn through *C*, *D*, and *E* if the side length of the square is *a*. (A. Savin)

***Quantum* Solution.** The answer is *a*, which becomes obvious after we shift the triangle downward by *a* (Figure 5).

So the solution offered in this version coincides with my solution above.

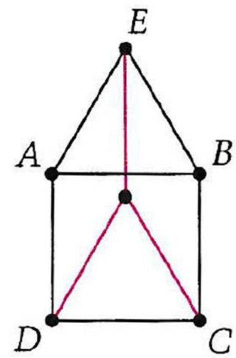


Figure 5

References

[1] "Brainteasers" B307 *Quantum Magazine*, Vol.11, No.2, National Science Teachers Assoc., Springer-Verlag, Nov-Dec 2000. p.3

[2] "Brainteasers" B182 *Quantum Magazine*, Vol.7, No.1, National Science Teachers Assoc., Springer-Verlag, Sep-Oct 1996. p.10