Puzzle of the Purloined Papers

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Ian Stewart has a nice logic problem in his *Casebook of Mathematical Mysteries* ([1]), which includes a pastiche of Sherlock Holmes in the form of Herlock Soames and Dr. Watsup, along with brother Spycraft and nemesis Prof. Mogiarty.

"An important document was accidentally mislaid, and then stolen," Spycraft said. "It is essential to the security of the British Empire that it be recovered without delay. If it gets into the hands of our enemies, careers will be ruined and parts of the Empire may fall. Fortunately, a local constable caught a glimpse of the thief, enough to narrow it down to precisely one of four men."

"Petty thieves?"

"No, all four are gentlemen of high repute. Admiral Arbuthnot, Bishop Burlington, Captain Charlesworth, and Doctor Dashingham."

Soames sat bolt upright. "Mogiarty has a hand in this, then."

Not following his reasoning, I asked him to explain.

"All four are spies, Watsup. Working for Mogiarty."

"Then ... Spycraft must be engaged in counter-espionage!" I cried.

"Yes." He glanced at his brother. "But you did not hear that from me."

"Have these traitors been questioned?" I asked.

Spycraft handed me a dossier, and I read it aloud for Soames's benefit. "Under interrogation Arbuthnot said 'Burlington did it.' Burlington said 'Arbuthnot is lying.' Charlesworth said 'It was not I.' Dashingham said 'Arbuthnot did it.' That is all."

"Not quite all. We know from another source that exactly one of them was telling the truth."

"You have an informer in Mogiarty's inner circle, Spycraft?"

"We *had* an informer, Hemlock. He was garrotted with his own necktie before he could tell us the actual name. Very sad—it was an Old Etonian tie, totally ruined. However, all is not lost. If we can deduce who was the thief, we can obtain a search warrant and recover the document. All four men are being watched; they will have no opportunity to pass the document to Mogiarty. But our hands are tied; we must stick to the letter of the law. Moreover, if we raid the wrong premises, Mogiarty's lawyers will publicise the mistake and cause irreparable damage."

Which man was the thief?

My Solution

A solution is easier to see in this case if a little symbolic logic is applied. We only use a few symbols. The equivalence notation is represented by "≡", or "⇔", or "iff", where A ≡ B means statement A is true if and only if (iff) statement B is true, or if A is true, then B is true, and if B is true, then A is true. The logical "and" is given by "∧" and means A∧B is true if and only if both A

and B are true; it is false if either or both A and B are false. Finally, negation is given by "~" where ~A means "not" A, that is, ~A is true if and only if A is false, and is false if A is true. (See the previous logic puzzles[1] for more details.)

Given this preamble, we label statements given in the story as follows:

A ≡ Arbuthnot is telling the truth   A' ≡ Arbuthnot stole the document.

B ≡ Burlington is telling the truth   B' ≡ Burlington stole the document.

C ≡ Charlesworth is telling the truth   C' ≡ Charlesworth stole the document.

D ≡ Dashington is telling the truth   D' ≡ Dashington stole the document.

Then the claims by the suspects can be translated into symbolic form as shown in the following table:

Statements	Symbolic Logic Translation
1 *Arbuthnot* says "Burlington did it."	A ⟺ ~A'∧B'∧~C'∧~D'
2 *Burlington* says "Arbuthnot is lying."	B ⟺ ~A
3 *Charlesworth* says "It was not I."	C ⟺ ~C'
4 *Dashingham* says "Arbuthnot did it."	D ⟺ A'∧~B'∧~C'∧~D'
5 Exactly one of them was telling the truth.	S ≡ Exactly one of A, B, C, D is true.

If A is true, then B is false (2) and ~C' is true (1), which means C is true (3). ∴ S is false (5). X

Suppose C is true. But B is false iff A is true, and A is false iff B is true (2). ∴ S is false (5). X

If D is true, then A' is true (4), and thus A is false (1), which implies B is true (2). ∴ S is false (5). X

Therefore A, C, and D are false. So A false implies B is true (2) (and therefore the only true statement among them), which implies S is true (5). Therefore no contradictions. But C false implies ~C' false (3). ∴ C' is true, that is, Charlesworth stole the document.

Stewart's Solution

"Charlesworth was the thief," said Soames.

"Are you certain, Hemlock? Much hangs upon your being right."

"There can be no doubt, Spycraft. Their statements are:

Arbuthnot: Burlington did it.

Burlington: Arbuthnot is lying.

Charlesworth: It was not I.

Dashingham: Arbuthnot did it.

We know that one of the men speaks truly and the other three lie. There are four possibilities. Let us try them in turn.

"If only Arbuthnot is telling the truth then his statement informs us that Burlington is the guilty

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party. However, Charlesworth is lying, so it was Charlesworth. This is a logical contradiction, so Arbuthnot is not telling the truth.

"If only Burlington is telling the truth then—"

"Charlesworth is lying!" I cried. "So it *was* Charlesworth!"

Soames glared at me for stealing his thunder. "That is so, Watsup, and the other statements are consistent with it. So we already know that Charlesworth is the thief. However, it is worth checking the other two possibilities to avoid even the remote possibility of error."

"Absolutely, old chap," said I.

He took out his pipe but did not light it. "If only Charlesworth is telling the truth then Burlington's statement is false, so Arbuthnot is telling the truth, again a contradiction since he is lying.

"If only Dashingham is telling the truth then the same contradiction arises.

"So the only possibility is that Burlington is the sole person telling the truth, and that confirms that the thief is Charlesworth. As Watsup so astutely deduced."

References

[1] Stewart, Ian, "The Puzzle of the Purloined Papers," *Professor Stewart's Casebook of Mathematical Mysteries*, Basic Books, 2014. p.77

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