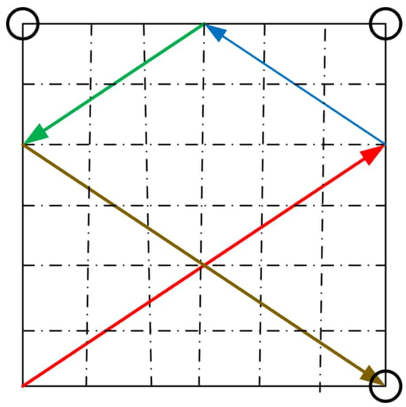
Snooker Puzzle

7 June 2021

Jim Stevenson



This is a nice puzzle ([1]) from Alex Bellos's Monday Puzzle column in the *Guardian*.

My cultural highlight of recent weeks has been the brilliant BBC documentary *Gods of Snooker*,[1, 2] about the time in the 1980s when the sport was a national obsession. Today's puzzle describes a shot to malfunction the Romford Robot … and put the Whirlwind … in a spin.

Baize theorem[3]

A square snooker table has three corner pockets, as [shown]. A ball is placed at the remaining corner (bottom left). Show that there is no way you can hit the ball so that it returns to its starting position.

The arrows represent one possible shot and how it would rebound around the table.

The table is a mathematical one, which means friction, damping, spin and napping do not exist. In other words, when the ball is hit, it moves in a straight line. The ball changes direction when it bounces off a cushion, with the outgoing angle equal to the incoming angle. The ball and the pockets are infinitely small (i.e. are points), and the ball does not lose momentum, so that its path can include any number of cushion bounces.

Thanks to Dr Pierre Chardaire, associate professor of computing science at the University of East Anglia, who devised today's puzzle.

My Solution

The approach uses the same idea as in my post Pool Party[4] for handling a pool ball bouncing off the side cushions.

Figure 1 shows the ball leaving the cushion at the same angle α that it arrived. If we reflect the table about the cushion, then the reflected image of the departing path makes the same angle α with the cushion. Since the total angles involved in the original bounce of the ball equaled 180° ($\alpha + \beta + \alpha$), so will the reflected image ($\beta + \alpha + \alpha$), which means the reflected image makes a straight line with the incoming path.

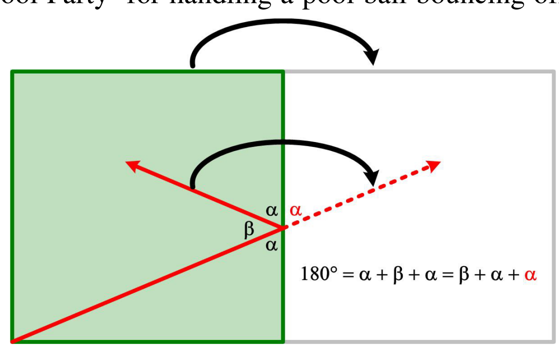


Figure 1

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1 JOS: The main difference between snooker and pool appears to be the number of balls: snooker uses 22 balls including one cue ball used to hit the others, whereas pool uses 15 balls plus a cue ball. (See *Wikipedia*)

2 https://www.bbc.co.uk/iplayer/episodes/m000w0pv/gods-of-snooker

3 JOS: I am not sure what this refers to. It seems to be: "*Baize Theorem is a blog of snooker analysis, written by Jack Webb. All blog posts can be found at medium.com/@baizetheorem/* …."

4 http://josmfs.net/2019/01/26/pool-party/

Figure 2 shows the situation after one bounce. We can parameterize the path of the ball using a coordinate system with origin (0, 0) at the starting point for the ball and unit-length defined by the side of the snooker table. Then the endpoint for the ball is at some coordinates (x, y). We can represent the straight-line path of the ball to this point by a vector from the origin given by

$$\mathbf{v} = x\,\mathbf{i} + y\mathbf{j}$$

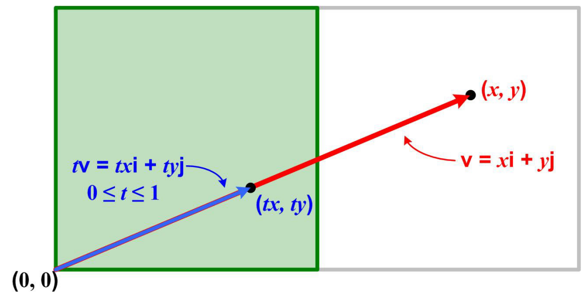

Figure 2

Then any point along the vector (line) to (x, y) can be given by

$$t\mathbf{v} = tx\,\mathbf{i} + ty\,\mathbf{j} = (tx, ty)$$

where $0 \leq t \leq 1$ (Figure 2).

In order to handle multiple bounces off other cushions, replicate the reflections of the snooker table around all the cushions of the original table and then around all the cushions of the reflected tables, *ad infinitum* (Figure 3). Using the above coordinate system based on a unit square for the snooker table, all the pockets of the table are reflected to corners of the reflected squares and so have integral coordinates. The original start point for the ball is also reflected to an infinity of corners, also with integral coordinates. Now the critical observation is that both of the coordinates of the start point corners have *even* values. But one or both of the coordinates of the pocket corners have *odd* values (Figure 3).

For a ball to bounce back to its starting point, its straight-line path through the reflected images must terminate at a reflected image of the start point and so this end point must have even coordinates. Divide these two even values by 2 (this is tantamount to multiplying the corresponding vector by $t = ½$ (see Figure 2)). If the numbers in the resulting new pair are also even, divide by 2 again (equivalently, multiply the original vector by $t = (½)^2$). We keep going in this way until one or

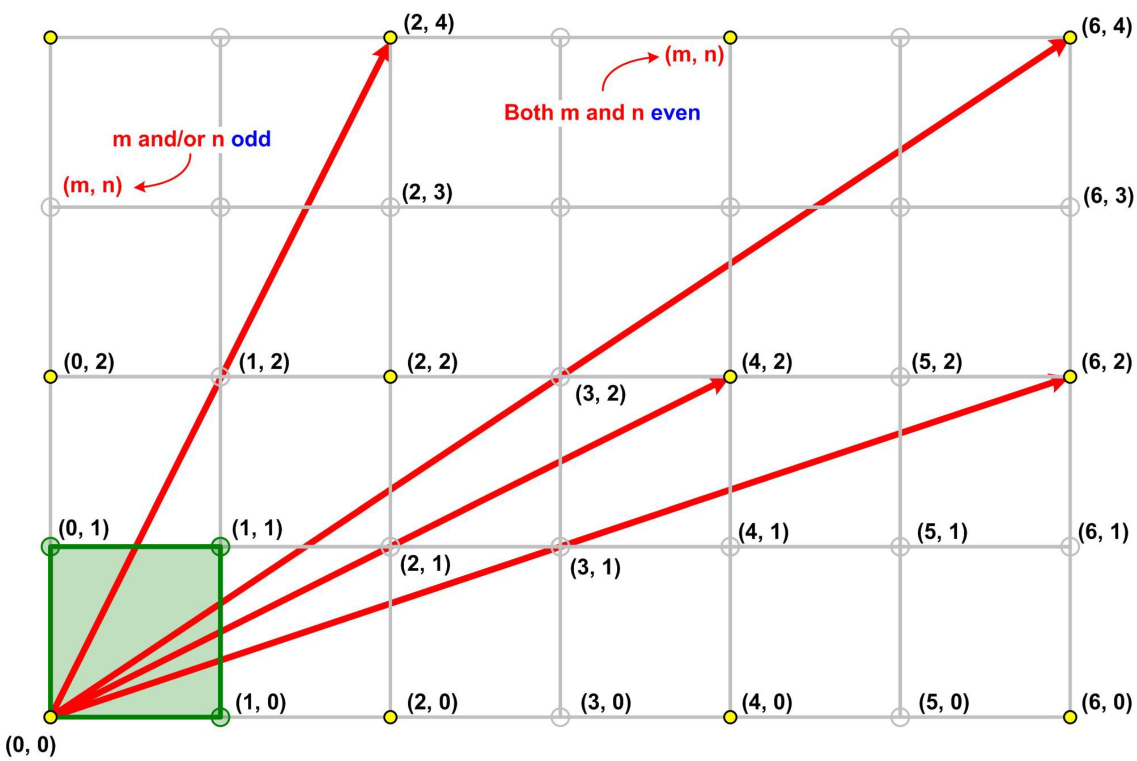

Figure 3

both of the two resulting coordinates is odd. This means we have landed on a reflected pocket corner along the vector to the reflected starting point for some $t = (½)^n$. (Notice that continuing to divide by 2 cannot result in both of the values being 0, for $x / 2^n = 0$ and $y / 2^n = 0$ means $x = 0·2^n = 0$ and $y = 0·2^n = 0$, so the only way to arrive at (0, 0) is to start there.)

Thus every attempt to send the ball on a path that ricochets back to the starting point will pass through a corner pocket first and be stopped from reaching its goal.

Bellos Solution

His solution is essentially the one I presented, only with a little less detail.

STEP 1 When a ball rebounds off a cushion, its ingoing and outgoing angles are the same. Thus if we were to consider the path of the ball continuing in the mirror image of the table, the path of the ball is a straight line, as illustrated [in Figure 4].

Each time the ball hits a cushion it continues into the mirror image of that table. By constructing mirror images of mirror images we get a grid of mirror images, and the straight line path of the ball continues ad infinitum (unless it falls in a pocket.)

STEP 2 [Figure 5] is a full grid of mirror images. Each square represents a table, and the red dots are the corners with no pocket. (Every other intersection has a pocket.) The only way to return the ball to its initial position would be to follow a path that is associated with a line segment that connects the bottom left corner of the grid to a red dot and does not contain any other red dots. However, as both coordinates of a red dot must be even numbers, the midpoint of such a line segment will be a grid point that corresponds to a pocket of the billiard table. Hence, it is not possible to return the ball to its initial position.

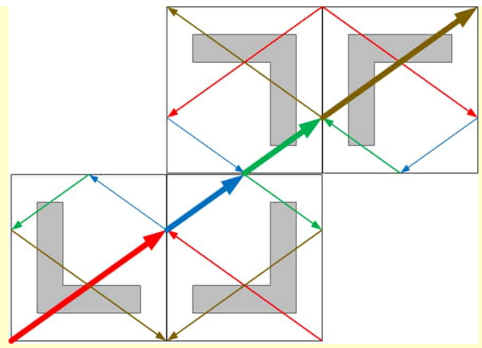

Figure 4

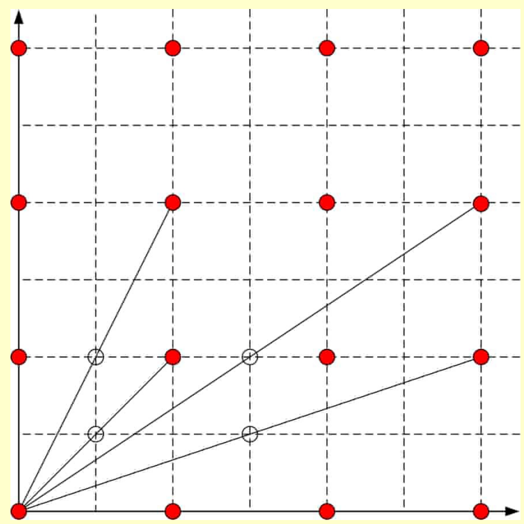

Figure 5

Bellos's argument eliminates my case where dividing both coordinates by 2 still leaves even numbers, since that would mean we landed on another reflected image of the start point along the line. This makes sense at first and simplifies the solution.

On the other hand, how would one know if "a line segment that connects the bottom left corner of the grid to a red dot and does not contain any other red dots", say for a dot that is very far from the lower left corner? The answer that comes to mind is to see if dividing by 2 the even coordinates associated with the particular red dot leads to one of the coordinates being odd. If not, keep dividing by 2 until that happens. In other words, follow the method I proposed. My method, at least, makes it clear that there are *no* possible red dots *anywhere* that could be joined to the lower left corner without the line passing through a pocket corner.

References

[1] Bellos, Alex, "Can you solve it? Gods of snooker", *Alex Bellos's Monday puzzle* , 31 May 2021 (https://www.theguardian.com/science/2021/may/31/can-you-solve-it-gods-of-snooker)