Old Hook Puzzle

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Here is another, more challenging, problem from the Sherlock Holmes puzzle book by Dr. Watson (aka Tim Dedopulos) ([1] p.142).

An event that occurred during *The Adventure of the Wandering Bishops* inspired Holmes to devise a particularly tricky little mental exercise for my ongoing improvement. There were times when I thoroughly appreciated and enjoyed his efforts, and times when I found them somewhat unwelcome. I'm afraid that this was one of the latter occasions. It had been a bad week.

"Picture three farmers," Holmes told me. "Hooklanders. We'll call them Ern, Ted, and Hob."

"If I must," I muttered.

"It will help," Holmes replied. "Ern has a horse and cart, with an average speed of eight mph. Ted can walk just one mph, given his bad knee, and Hob is a little better at two mph, thanks to his back."

"A fine shower," I said. "Can't I imagine them somewhat fitter?"

"Together, these worthies want to go from Old Hook to Coreham, a journey of 40 miles. So Ern got Ted in his cart, drove him most of the way, and dropped him off to walk the rest. Then he went back to get Hob [who was still walking], and took him into Coreham, arriving exactly as Ted did. How long did the journey take?"

Can you find a solution?

I added the statement in brackets. I initially thought Hob waited in Old Hook until Ted fetched him. But the solution indicated that was not the case. So I realized Hob had started out at the same time the others left Old Hook. The solution has some hairy arithmetic. Even knowing the answer it is difficult to do the computations without a mistake.

My Solution

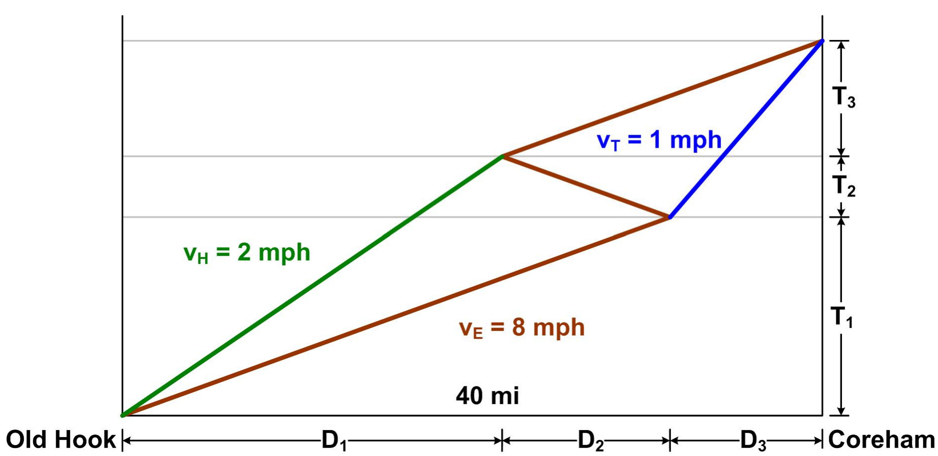

Figure 1 Problem Set Up

Figure 1 shows a space-time diagram of the problem. We wish to find the total time T where

$$T = T_1 + T_2 + T_3$$

Ted's speed is $v_T = 1$ mph, Hob's speed is $v_H = 2$ mph, and Ern's speed is $v_E = 8$ mph. Then we have the following 6 equations in 6 unknowns:

$$D_1 + D_2 + D_3 = 40 \tag{1}$$

$$v_E\, T_1 = D_1 + D_2$$

$$v_H\, (T_1 + T_2) = D_1$$

$$v_T\, (T_2 + T_3) = D_3$$

$$v_E\, T_2 = D_2$$

$$v_E\, T_3 = D_2 + D_3$$

or

$$8\, T_1 = D_1 + D_2 \tag{2}$$

$$2\, (T_1 + T_2) = D_1 \tag{3}$$

$$T_2 + T_3 = D_3 \tag{4}$$

$$8\, T_2 = D_2 \tag{5}$$

$$8\, T_3 = D_2 + D_3 \tag{6}$$

Equations (1), (2), (5), (6) imply

$$8\, T = 8\, (T_1 + T_2 + T_3) = D_1 + 3\, D_2 + D_3 = 40 + 2\, D_2$$

$$\boxed{T = 5 + (1/4)\, D_2} \tag{7}$$

Equations (1), (3), (4) imply

$$2\, T_1 + 3\, T_2 + T_3 = D_1 + D_3 = 40 - D_2$$

and with equation (5)

$$40 - D_2 = T_1 + 2\, T_2 + T = T_1 + (1/4)\, D_2 + T$$

so

$$\boxed{T + T_1 = 40 - (5/4)\, D_2} \tag{8}$$

Equation (4) implies

$$T - T_1 = D_3 \tag{9}$$

Equations (5), (6) imply

$$8\, (T_2 + T_3) = 2\, D_2 + D_3$$

so with (9)

$$8\, (T - T_1) = 2\, D_2 + (T - T_1)$$

Therefore

$$\boxed{T - T_1 = (2/7)\, D_2} \tag{10}$$

Adding equations (8) and (10)

$$2\, T = 40 - (27/28)D_2$$

or

$$\boxed{T = 20 - (27/56)\ D_2} \qquad (11)$$

Subtracting equations (7) and (11) yields

$$D_2 = 840/41 = 20 + (20/41)$$

Plugging this into equation (7) yields

$$T = 5 + (1/4)(20 + (20/41)) = 10 + 5/41 \text{ hours}$$

Dr. Watson's Solution

The answer is 10 and 5/41sts hours. If Ted rides for x miles at 8 mph, then his journey time =

$$(x/8) + (40 - x)/1.$$

Hob walks for y miles, so his journey time is

$$y/2 + (40 - y)/8.$$

This means that Ern's journey time is

$$x/8 + (x - y)/8 + (40 - y)/8.$$

Now all these total times are equal. So

$$(x/8) + 40 - x = y/2 + (40 - y)/8,$$

which means

$$7x + 3y = 280.$$

Also, by multiplying the second and third equation by 8,

$$4y + 40 - y = x + x - y + 40 - y,$$

and

$$2x - 5y = 0.$$

So now we have two simple equations for x and y. Solve, and we'll find that

$$x = 1400/41, \text{ and } y = 560/41.$$

Note that leaving it in terms of 1/41 is simplest for this solution. Substitute x into Ted's time or y into Hob's time, and you'll find that the total is 10 and 5/41 hours.

Dr. Watson had a cleaner path through the equations by obtaining 3 equations for the total time T corresponding to each of the three men. I just proceeded mindlessly through the algebra.

References

[1] Dedopulos, Tim, *The Sherlock Holmes Puzzle Collection: The Lost Cases*, Metro Books, Sterling Publishing Co., New York, Carlton Books Ltd., London, 2015.