The Hose Knows

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This is a fairly straight-forward Brainteaser from the *Quantum* magazine ([1]).

A man is filling two tanks with water using two hoses. The first hose delivers water at the rate of 2.9 liters per minute, the second at a rate of 8.7 liters per minute. When the smaller tank is half full, he switches hoses. He keeps filling the tanks, and they both fill up completely at the same moment. What is the volume of the larger tank if the volume of the smaller tank is 12.6 liters?

My Solution

The first thing to notice is the missing information, namely, we don't know which hose fills the smaller tank first. Hopefully, it will not matter.

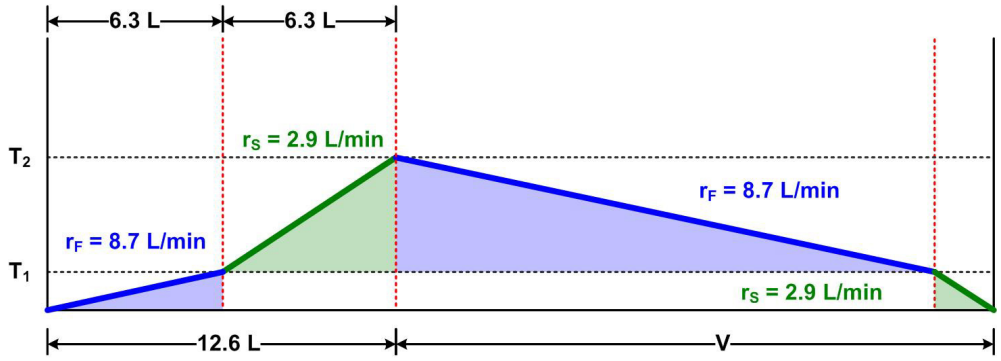

Figure 1 Fast Fill of Smaller Tank First

Figure 1 shows a "space-time diagram" of the problem assuming the fast hose fills the smaller tank first. Instead of "distance," we have "volume" as the horizontal axis. The parameters are

r_F = 8.7 L/min (fast hose)

r_S = 2.9 L/min (slow hose)

T_1 = time to fill the smaller tank half-way

T_2 = time to fill both tanks

V = volume of large tank

12.6 L = volume of smaller tank

Then we have the following relations

$$6.3\ L = r_F\, T_1 = 8.7\, T_1$$

$$6.3\ L = r_S\, (T_2 - T_1) = 2.9\, (T_2 - T_1)$$

and

$$V = r_S\, T_1 + r_F\, (T_2 - T_1) = 2.9\, T_1 + 8.7\, (T_2 - T_1) = 2.9 \times 6.3/8.7 + 8.7 \times 6.3/2.9 = 6.3(1/3 + 3) = 21\ L$$

Now to consider what happens if we fill the smaller tank first with the slower hose. Figure 2 shows the situation. We see that the hoses fill the same amounts in the same amounts of time in each

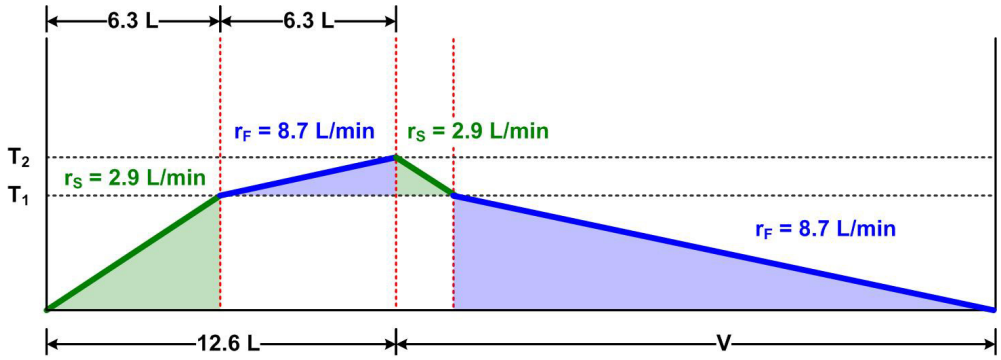

Figure 2 Slow Fill of Smaller Tank First

tank as before, only in a different order. So it doesn't matter which hose is used to fill the smaller tank first.

Quantum Solution

There are two possible cases: (a) the smaller tank is filled first from the less powerful hose, or (b) it is filled first from the more powerful hose. Let us work with case (a) first. In this case, it will take 6.3/2.9 minutes to fill the first half of the small container, during which the larger container receives (6.3/2.9)(8.7) liters of water. Then it will take 6.3/8.7 minutes to fill the second half of the small container, and the same amount to top off the larger one (because the jobs are completed at the same time). So the larger container, during this time, receives (6.3/8.7)(2.9) liters of water. Since this fills the larger container, it must contain

$$(6.3/2.9)(8.7) + (6.3/8.7)(2.9) = 21 \text{ liters.}$$

Case (b) is left for the reader, who may notice that it requires exactly the same arithmetic as case (a).

References

[1] "Brainteasers" *Quantum* Vol.10, No.5, National Science Teachers Assoc., Springer-Verlag, May-Jun 2000. p.3 B293