# Catching the Thief

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This typical problem from the prolific H. E. Dudeney ([1]) may be a bit tricky at first.

104.—CATCHING THE THIEF.

"Now, constable," said the defendant's counsel in cross-examination," you say that the prisoner was exactly twenty-seven steps ahead of you when you started to run after him?"

"Yes, sir."

"And you swear that he takes eight steps to your five?"

"That is so."

"Then I ask you, constable, as an intelligent man, to explain how you ever caught him, if that is the case?"

"Well, you see, I have got a longer stride. In fact, two of my steps are equal in length to five of the prisoner's. If you work it out, you will find that the number of steps I required would bring me exactly to the spot where I captured him."

Here the foreman of the jury asked for a few minutes to figure out the number of steps the constable must have taken. Can you also say how many steps the officer needed to catch the thief?

## My Solution

The information seemed a bit confusing at first, so I carefully translated the problem into algebraic statements and hoped the answer would fall out. It did.

Define the following:

| | |
|---|---|
| $r_T$ = thief steps per unit time | $L_T$ = thief step length |
| $r_C$ = constable steps per unit time | $L_C$ = constable step length |
| and $r_C / r_T = 5/8$ | and $2L_C = 5L_T$ |

Then Figure 1 depicts the chase where the thief has a head start of 27 steps,[1] and the constable catches him after some time t. This yields the equation

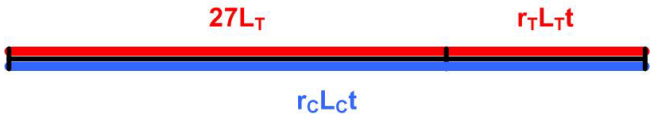


**Figure 1**

$$27L_T + r_T L_T t = r_C L_C t$$

Then

$$27 + r_T t = r_C (L_C/L_T) t$$

$$27 + r_C (8/5) t = r_C (5/2) t$$

$$27 = (5/2 - 8/5) r_C t = (9/10) r_C t$$

[1] JOS: We are assuming the "steps" here refers to the thief's steps with their corresponding length.

$$30 = r_C\, t$$

Therefore, in the time t to catch the thief, the constable takes 30 steps.

It is nice that we don't have to determine the actual length of the steps and the time to catch the thief in order to solve the problem.

## Dudeney Solution

The constable took thirty steps. In the same time the thief would take forty-eight, which, added to his start of twenty-seven, carried him seventy-five steps. This distance would be exactly equal to thirty steps of the constable.

## References

[1] Dudeney, Henry Ernest, *Amusements In Mathematics*, 1917