

Serious Series

17 December 2019 Jim Stevenson

If s_n denotes the sum of the first n natural numbers, find the sum of the infinite series

$$\frac{s_1}{1} + \frac{s_2}{2} + \frac{s_3}{4} + \frac{s_4}{8} + \dots$$

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Solution (Evander)

S_n denotes the sum of the first n natural numbers.

example: $S_1 = 1$; $S_2 = 1+2 = 3$; $S_3 = 1+2+3 = 6$

and: $S_n = 1+2+3+\dots+n$

$$\text{So, } \frac{s_1}{1} + \frac{s_2}{2} + \frac{s_3}{4} + \frac{s_4}{8} + \frac{s_5}{16} + \dots = \frac{1}{1} + \frac{3}{2} + \frac{6}{4} + \frac{10}{8} + \frac{15}{16} + \dots$$

$$\text{Let, } A = 1 + \frac{3}{2} + \frac{6}{4} + \frac{10}{8} + \frac{15}{16} + \dots$$

$$2A = 2 + 3 + \frac{6}{2} + \frac{10}{4} + \frac{15}{8} + \frac{21}{16} + \dots \quad (-)$$

$$A = 4 + \frac{3}{2} + \frac{4}{4} + \frac{5}{8} + \frac{6}{16} + \dots$$

$$A = \left\{ \begin{array}{l} 4 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \\ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \\ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \\ \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \\ \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \\ \vdots \end{array} \right.$$

$$A = 4 + \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right) + \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right) + \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right) + \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \right) + \left(\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \right) + \dots$$

$$= 4 + 2 \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right) + \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right) + \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \right) + \left(\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \right) + \dots$$

$$= 4 + 2 \cdot \left(\frac{\frac{1}{2}}{1 - \frac{1}{2}} \right) + \left(\frac{\frac{1}{2}}{1 - \frac{1}{2}} \right) + \left(\frac{\frac{1}{4}}{1 - \frac{1}{2}} \right) + \left(\frac{\frac{1}{8}}{1 - \frac{1}{2}} \right) + \dots$$

$$= 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \frac{4}{1 - \frac{1}{2}} = \frac{4}{\frac{1}{2}} = \underline{\underline{8}}$$