Loggers Problem

28 March 2020

Jim Stevenson

Here is another delightful problem from the Sherlock Holmes puzzle book by Dr. Watson (aka Tim Dedopulos) ([1] p.94).

In Sussex, Holmes and I ran into a pair of woodcutters named Doug and Dave. There was an air of the unreliable about them—not helped by a clearly discernable aroma of scrumpy—but they nevertheless proved extremely helpful in guiding us to a particular hilltop clearing some distance outside of the town of Arundel. A shadowy group had been counterfeiting sorceries of a positively medieval kind, and all sorts of nastiness had ensued.

The Adventure of *the Black Alchemist* is not one that I would feel comfortable recounting, and if my life never drags me back to Chanctonbury Ring I shall be a happy man. But there is still some instructive material here. Whilst we were ascending our hill, Doug and Dave made conversation by telling us about their trade. According to these worthies, working together they were able to saw 600 cubic feet of wood into large logs over the course of a day, or split as much as 900 cubic feet of logs into chunks of firewood.

Holmes immediately suggested that they saw as much wood in the first part of the day as they would need in order to finish splitting it at the end of the day. It naturally fell to me to calculate precisely how much wood that would be.

Can you find the answer?

Solution

I proceed by translating the problem into algebraic statements. Let $r_L = 600\ \text{ft}^3/\text{day}$ be the volume of wood obtained each day by sawing logs and let $r_F = 900\ \text{ft}^3/\text{day}$ be the volume of wood obtained each day by splitting logs into firewood. Let T be the fraction of the day devoted to sawing lumber and 1 – T the fraction devoted to chopping firewood. Then we want these two activities to result in the same volume of wood, that is, we want

$$r_L\, T = r_F\, (1 - T)$$

or

$$600\, T = 900\, (1 - T)$$

or

$$2T = 3(1 - T).$$

So

$$T = 3/5 \text{ of a day for sawing logs}$$

Therefore, the volume of logs sawed, which would be equal to the volume of firewood chopped, would be

$$r_L\, T = 600\, (3/5) = 360 \text{ cubic feet}$$

Dr. Watson's Solution

[The answer is] 360 cubic feet of wood, which works out at a little over 2.8 cords. The relative amount of time Doug and Dave spend sawing and splitting must match the ratio of how much of each job they can perform in one day—that is, they must divide their day by 6:9, or 2:3. Thus they must spend 3/5ths of the day sawing, which is slower, and 2/5ths of the day splitting. 3/5ths of 600 is 360 cubic feet of wood (as is 2/5ths of 900).

References

[1] Dedopulos, Tim, *The Sherlock Holmes Puzzle Collection: The Lost Cases*, Metro Books, Sterling Publishing Co., New York, Carlton Books Ltd., London, 2015.