Barrier Minimal Path Problem

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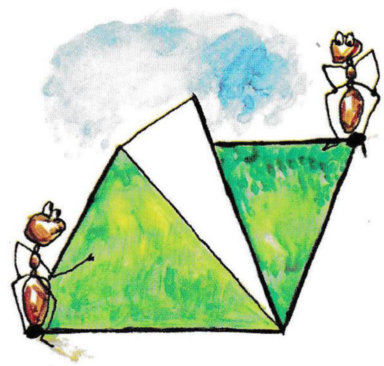
Jim Stevenson



Pavel Chernusky

This is a nifty little problem from the *Quantum* math magazine ([1]).

Two ants stand at opposite corners of a 1-meter square. A barrier was placed between them in the form of half a 1-meter square attached along the diagonal of the first square, as shown in the picture. One ant wants to walk to the other. How long is the shortest path?

My Solution

I imagined the first ant laying the barrier flat and thus forming a square. I also imagined the second ant doing the same thing on his side. Then I pasted an instance of the second ant's square along the first ant's square, matching the right edge of the first ant's square with the left edge of the second ant's square (Figure 1). I then pasted a second instance of the second ant's square along the first ant's, this time matching the left edge of the first ant's square with the second ant's right edge. (These operations reminded me of the Penrose diagrams for gluing two different universes—or two different regions of the same universe—along a wormhole using the Kruskal coordinate system.)

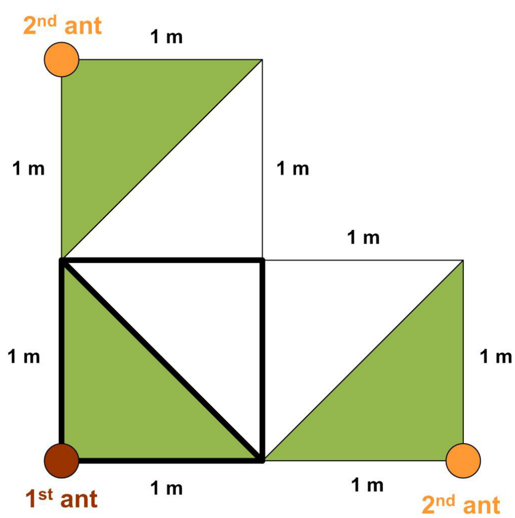


Figure 1 "Penrose Diagram" for Problem

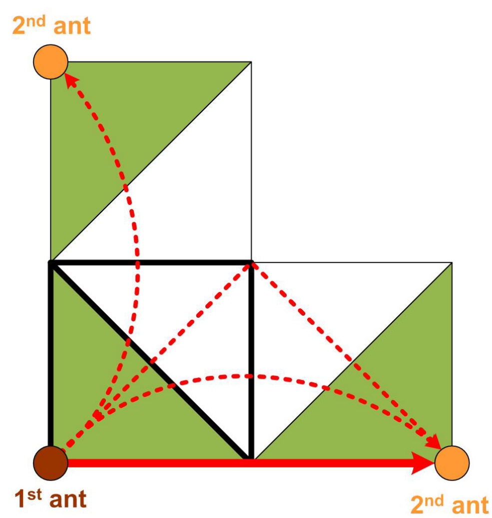


Figure 2 Shortest Path Solution

So if the first ant wants to reach the second ant and he wanders up and over the barrier along its right edge, he will come down over the left edge of the second ant's side of the barrier. And similarly if the first ant goes over the left edge of the barrier, he will come down the right edge on the second ant's side. Figure 2 shows examples of these paths, together with one straight over the vertex of the barrier. It is clear in all cases the shortest path is one along the right or left edge of the "squares" as shown by the red arrow in Figure 2—that is, a path that goes around the barrier. Thus the shortest path is 2 meters long.

Quantum Solution

The Quantum solution argues a bit less visually for the same solution.

Suppose one ant, in walking toward the other, tries to go toward the barrier rather than around it (along the edges of the square). Certainly, its path should be symmetric with respect to the barrier: if it follows two different paths, then one must be shorter than the other, and the longer path wastes time. Also, when it gets to the base of the barrier, the shortest way over is a path perpendicular to an edge that is not on the ground. If we fold the barrier flat against the original square, we get the diagram in Figure 3. Since ABCD is a square, we are comparing a + b to b + c. Since a > c, the path along the edge of the original square is the shortest.

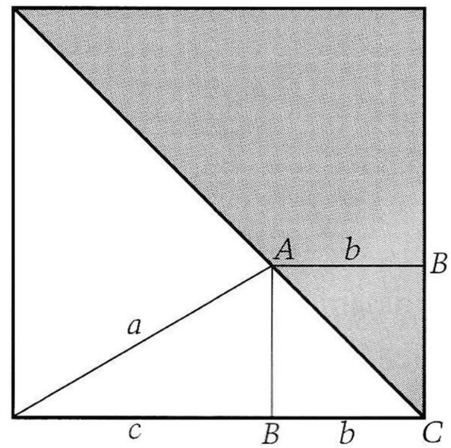


Figure 3 *Quantum* Solution

References

[1] "Traveling Ants" B200 "Brainteasers" *Quantum Magazine*, Vol.7, No.4, National Science Teachers Assoc., Springer-Verlag, Mar-Apr 1997. p.17