Square Deal

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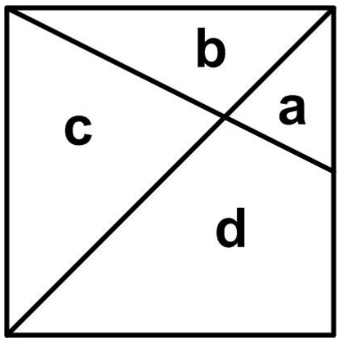
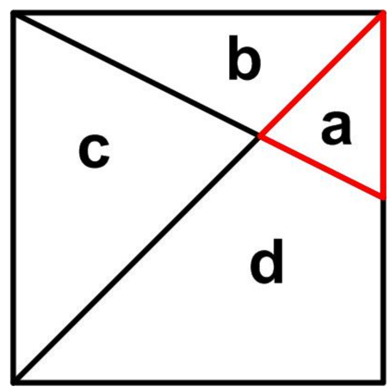


Here is a simple *Futility Closet* problem from 2014 ([1]).

This unit square is divided into four regions by a diagonal and a line that connects a vertex to the midpoint of an opposite side. What are the areas of the four regions?

My Solution



The unit square has area 1. Then we have the following relationships for the areas inside the square:

$$\mathbf{a} + \mathbf{b} = \tfrac{1}{4} \quad (1)$$

$$\mathbf{b} + \mathbf{c} = \tfrac{1}{2} \quad (2)$$

$$\mathbf{a} + \mathbf{d} = \tfrac{1}{2} \quad (3)$$

We need one more relationship. Notice that the triangle **a** is similar to triangle **c**, since they have equal angles. But the base of **a** is ½ the base of **c**, so that all dimensions of **a** are ½ of **c**. This means the area **a** = ¼ **c** or

$$\mathbf{c} = 4\,\mathbf{a}. \quad (4)$$

Therefore substituting equation (4) into equation (2) and subtracting equation (1) from equation (2) yields **a** = 1/12. Therefore **c** = 1/3, **b** = 1/6, and **d** = 5/12. My solution (worked out before looking at Futility Closet's) agrees with Futility Closet's solution.

Futility Closet Solution

a + b = 1/4, b + c = 1/2, and a + d = 1/2. Triangles a and c are similar, and c has twice the linear dimensions of a, so c = 4a. That's enough to work out the areas: a = 1/12, b = 1/6, c = 1/3, and d = 5/12.

From University of Toronto mathematician Ed Barbeau's *After Math* (1995).

References

[1] "Square Deal" *Futility Closet*, 15 December 2014 (https://www.futilitycloset.com/2014/12/15/square-deal-4/, retrieved 6/19/2015)