Down With Geometry

13 November 2020

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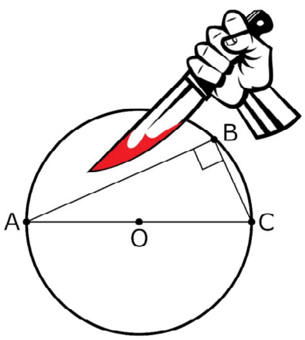

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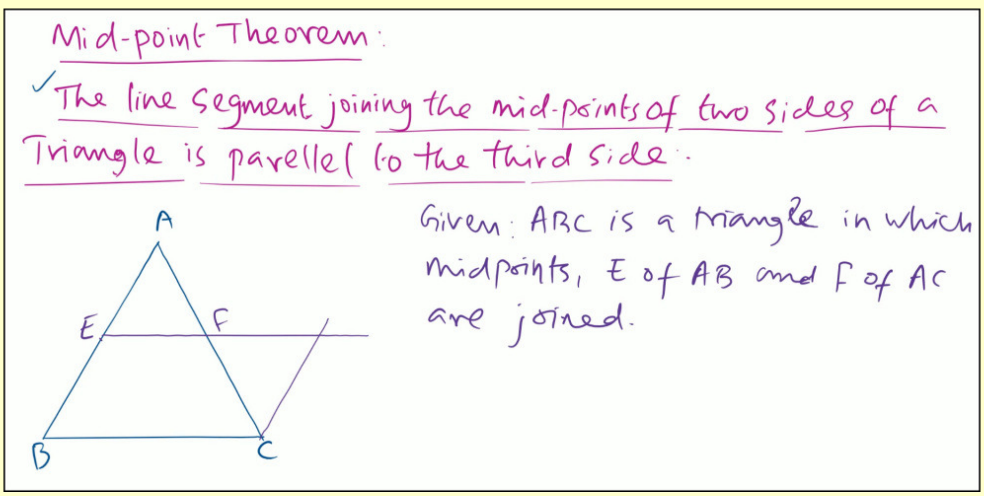
One of my favorite bloggers, Kevin Drum, decided to relieve the tedium of our current political anarchy by whacking the hornets' nest of the high school mathematics curriculum, in particular the subject of plane geometry. You can tell from the tag list on my blog that I hold plane geometry in high regard and can't let this gibe pass without some rebuttal, futile as it may be. Actually, I am not going to weigh in on the general issue of the current math curriculum that much, but rather make a few observations from my own experience over the years as it relates to Kevin's post ([1]).

First, the post:

Down With Geometry!

Kevin Drum, 11 November 2020



Sure, I guess. But who cares?

Everything seems to be pretty calm in the political world these days, so let's switch gears momentarily and talk instead about math education. According to the *Wall Street Journal*, Steven Levitt (of *Freakonomics* fame) is now the one-billionth person to propose that we change the high school math curriculum to include more "useful" subjects:[1]

> Dr. Levitt's proposal is simple: Condense three years of high-school math—typically Algebra I in ninth grade, Geometry in 10th grade and Algebra II in junior year—to two years. Then, devote the freed up time to more relevant learning, such as data science or financial literacy.

[1] https://www.wsj.com/articles/the-movement-to-modernize-math-class-11605110629?mod=hp_listc_pos2

I have a better idea: just get rid of plane geometry completely. It is entirely useless both on its own and as a steppingstone to further studies in math, which are almost all based on algebra and analytic geometry.

But Kevin! Geometry is where we lean about axioms and proofs. Those are critical to understanding how math works.

No, they aren't. But if you really want everyone to learn about the foundations of math, then make it a one-month unit in Algebra II or something. You can use plane geometry as an example, or you can use arithmetic, which is probably a better bet. Or you can just skip it, since foundations is a fairly advanced subject that's of no real use for anything at the high school or undergrad level.[2] As for proofs, those are already covered in a semi-intuitive way in Algebra I, and that's plenty. Students will get plenty of proof workouts later on.

This would have the added benefit of making Algebra I and Algebra II into consecutive courses, instead of giving students a full year to forget Algebra I before they take up their study of more advanced subjects. A condensed unit on trigonometry, which *is* semi-useful, could be folded into Algebra II, taking the place of worthless topics like synthetic division. This can be followed by either calculus or data science, depending on your druthers.

Anyway, down with geometry! Who's with me?

It looks like both Drum's and Levitt's position boils down to the perennial utility argument: is a high school subject "useful". The follow-up question, of course, is "useful for what?". And that is where things get complicated. The first fork in the road is whether or not a student will pursue a future in science or technology. If not, mathematicians such as Keith Devlin[3] and David Bressoud[4] at MAA's Math Values website have been posting numerous comments and studies about what mathematics would be appropriate ("useful") for non-technical track students. They have also been addressing the issue of what is appropriate for technical students as well. For the technically motivated students it seems that the CORE program wants to push more traditionally advanced mathematics down the educational ladder into the high school and even grade school curriculum in order to be competitive with the heavily technological education of other countries. So clearly "useless" plane geometry is a prime candidate for pruning.

I demur. I am an old, died-in-the-wool, liberal arts fogy. To me education is more than training, though public education has an obligation to provide basic life supporting skills for its population ("reading, writing, 'rithmetic"). But in a democracy all citizens need to have some understanding of both technology and its underpinnings and the humanities, such as history, geography, languages, and the arts—namely, Culture. Where would a case for plane geometry fit in that scheme?

Consider music. What good is it? What is its *use*? Certainly not as a path to a profession. Only a tiny minority can succeed in that endeavor. So why do humans require music in their lives? We just do, we really do. Very few can become composers, but many can learn to play an instrument or master singing, and even more, in fact all, can enjoy listening to music. Another human foible is puzzles and games. We love the mental challenge involved in these pastimes. There is something

[2] I say this as someone with a considerable fondness for the foundations of math. Why else would I have a cat named Hilbert?

[3] https://mathvalues.squarespace.com/devlinsangle [JOS: provocative author of the sadly titled "All The Mathematical Methods I Learned In My University Math Degree Became Obsolete In My Lifetime" (https://www.huffingtonpost.com/entry/all-the-mathematical-methods-i-learned-in-my-university_us_58693ef9e4b014e7c72ee248?ncid=engmodushpmg00000004), as well as other iconoclastic articles that I hope someday to address.]

[4] https://mathvalues.squarespace.com/launchings

deeply pleasurable in solving a puzzle or executing a strategy in a game. Akin to that is the lure of mysteries—in history, in nature, about the future.

As a kid, I loved all these things—jig-saw puzzles, hidden pictures, word searches, codes, riddles, and the TV show *This Is the Story*, which recounted surprising episodes from history in 15-minute sequences of pen and ink drawings (for example, the origin of the success of the new Pullman railroad coach—it was used for Lincoln's funeral train back to Illinois and required enlarging the tunnels through the Appalachians).

What I didn't like as a kid was math, that is, arithmetic! The memorization, the timed tests, the panic. In junior high we often had to essentially solve word problems before learning algebra. In high school we learned trigonometry in order to perform the eminently "useful" task of measuring the distance across swamps, all without the aid of hand calculators, but with those handy logarithm tables requiring our interpolation skills. Then, of course, there was the *pièce de résistance* of, wait for it—solid geometry! So I am not unacquainted with the need to cut out archaic deadwood from the school curriculum. (In our day, the fight was over whether to take Latin or not. I lost. I had to take Latin. I would have been much better off taking an additional two years of Spanish instead, though the meddling gods decided to send me to the one large country in South America that did not speak Spanish.)

So after all that refuse of early math torture why did I gravitate to math? *Plane geometry*. I have asked a number of my contemporary math friends through the years as to why they followed a math career and virtually all mentioned their exhilarating experience in plane geometry. It is so different from tedious calculations (and mistakes) and wonderfully visual. It truly reveals the essence of mathematics, which is puzzle-solving *par excellence*. This is clearly evident in the fantastic geometric puzzles of Catriona Agg (née Shearer) that have figured so prominently in this blog.

Contra Drum, plane geometry is also an eminently simple way to learn deductive reasoning, which is at the heart of mathematics and all reasoned discourse. Euclid's *Elements* certainly were essential for such leaders as Jefferson and Lincoln. As I mentioned in an earlier post[5] and is supported further in Kucharski's article "Euclid as Founding Father,"([2]), for Lincoln,

> Euclid's mathematical principles and system of logic would go on to become a crucial tool throughout his political career. Euclidean logic would shape his arguments against slavery, and his debates against its proponents. It would convince him of what was just, and what was unacceptable. As he rose to the presidency, and as the nation descended into civil war, Euclid would be there to guide him.

Again the rationalism of the Enlightenment that infused Jefferson's Declaration of Independence[6] depended not only on the idea of Newton's laws, but also on Euclid's *Elements*. In fact Newton employed this deductive method of reasoning for his monumental *Principia* rather than the methods of his new calculus just because it was the gold standard for rational discourse, and had been for some 2000 years. If for no other reason, a culturally educated citizen of America living under its democratic principles should be intimately aware of this history and its dependence on rationalism as the glue to bond a disparate populace—and that requires an experience with plane geometry.

But there is an even deeper, personal reason everyone should be exposed to the rigors of Euclidean deductive proofs. It is to confront the idea of "truth." Mathematics provides a system of truth based on logic whose simplest exemplar and progenitor is Euclid's geometry. This could just be an esoteric philosophical side issue except for the mysterious success mathematics has had in describing our physical reality, and, through the more recent development of probability and

[5] http://josmfs.net/2020/07/18/abraham-lincoln-technologist/
[6] http://josmfs.net/2019/10/24/newton-and-the-declaration-of-independence/

statistics, our social and psychological behavior. But do probabilistic and statistical "truths" share the same level of certainly as Euclidean geometry truths? When a poll gives a "margin of error" that we see in the recent election to be bogus, what are we to make of it? We are faced with a barrage of risk analyses regarding the Covid-19 infection and the efficacy of potential vaccines. How do we assess the veracity of the associated mathematical statements? Scientists using the concept of statistical "significance" are at such loggerheads over its meaning that they recommend abandoning it. These difficulties are all aspects of data science, which is being touted as a replacement for the standard math curriculum. I contend understanding the role of the math involved entails a fair amount of mathematical maturity that is not easily acquired at an elementary level. These considerations are all part of the bigger issue of how do we know what we know, that is, epistemology. A major part of any person's education should be developing the skills to assess "truth" (weigh evidence, seek confirmation, etc.), but it may take a lifetime to accomplish, if ever.

I have long struggled with the term "postmodernism." I was stymied by its evident oxymoron. But recently I have come to appreciate that it means there is no objective truth,[7] that everyone has their own narratives of what is so. The Bush-Cheney administration talked about "making their own reality" and Kellyanne Conway of the Trump administration came up with "alternative truths." Underlying this view is the notion of coercion—what is true is what the most powerful make to be true. Clearly, this rampant, coercive subjectivism is the opposite of rationalism.

The antidote is plane geometry!

So ends my panegyric to Euclid's monumental and magnificent plane geometry.

Kevin, I'm *not* with you!

References

[1] Drum, Kevin, "Down With Geometry!" *Kevin Drum* (blog), 11 November 2020 (https://www.motherjones.com/kevin-drum/2020/11/down-with-geometry/, retrieved 11/12/2020)

[2] Kucharski, Adam, "Euclid as Founding Father", *Nautilus*, October 13, 2016 (http://nautil.us/issue/41/selection/euclid-as-founding-father, retrieved 11/13/2020)

[7] https://en.wikipedia.org/wiki/Postmodernism