Swallowing Elephants

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This is a simple logic puzzle from one of Ian Stewart's many math collections ([1]).

1. Elephants always wear pink trousers.
2. Every creature that eats honey can play the bagpipes.
3. Anything that is easy to swallow eats honey.
4. No creature that wears pink trousers can play the bagpipes.

Therefore:

Elephants are easy to swallow.

Is the deduction correct, or not?

My Solution

We proceed as in the Pointing Fingers[1] logic puzzle by converting the statements to symbolic logic. Make the following statement assignments:

E = A creature is an elephant.

P = A creature wears pink trousers.

H = A creature eats honey.

B = A creature plays the bagpipes.

S = A creature is easy to swallow.

Now convert the statements in the puzzle to symbolic logic with these assignments, recalling that the implication $P \Rightarrow Q$ means "if P is true, then Q is true" and ~P is the logical negation of P.

1. $E \Rightarrow P$
2. $H \Rightarrow B$
3. $S \Rightarrow H$
4. $P \Rightarrow \sim B$

$\therefore\ E \Rightarrow S$

Now recall a couple of logic rules that will help in our reasoning, which I will couch in symbolic logic form. (Again note that ~A is the logical negation (or "not") of A, that is, if A is true, then ~A is false, and if A is false, then ~A is true.)

(Transitive Rule) If A, B, and C are three statements, and if $A \Rightarrow B$ and $B \Rightarrow C$, then $A \Rightarrow C$

(Contrapositive) If A and B are two statements, and if $A \Rightarrow B$, then $\sim B \Rightarrow \sim A$, which latter is called the *contrapositive*. Since $\sim(\sim P) \equiv P$, then $(A \Rightarrow B) \equiv (\sim B \Rightarrow \sim A)$, that is,

[1] http://josmfs.net/2020/09/19/pointing-fingers/

the implication and the contrapositive are logically equivalent—they are both true or both false, one is true if and only if the other is true.

(Converse) If A and B are two statements, and if A ⇒ B, then the implication B ⇒ A is called the *converse* (and is *only* true of A and B are logically equivalent, that is, A is true if and only if B is true)

In words, the contrapositive statement is if A is true means B is true, then if B is not true, then A must not be true either. Be careful not to confuse the contrapositive with the converse, which is not always true.

Now, reorder the 4 statements in "A ⇒ B and B ⇒ C and C ⇒ D …" chains as best as possible, substituting the contrapositive where necessary.

1. E ⇒ P

4. P ⇒ ~B

2. ~B ⇒ ~H (contrapositive of H ⇒ B)

3. ~H ⇒ ~S (contrapositive of S ⇒ H)

∴ E ⇒ ~S (by the transitive rule)

Therefore, E ⇒ S is not a correct deduction.

This solution turns out to be basically the same as Professor Stewart's, so I won't repeat it.

References

[1] Stewart, Ian, "Swallowing Elephants," *Professor Stewart's Hoard of Mathematical Treasures*, Profile Books Ltd., London, 2009 (Basic Books, 2010). p.9