Rufus Puzzle

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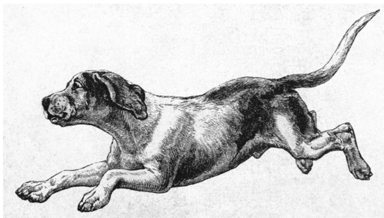

Again we have a puzzle from the Sherlock Holmes puzzle book by Dr. Watson (aka Tim Dedopulos) ([1] p.162). This one is quite a bit more challenging, at least for me.

When Holmes and I met with Wiggins one afternoon, he was accompanied by a rather scrappy-looking mutt, who eyed me with evident suspicion.

"This is Rufus," Wiggins said. "He's a friend."

"Charmed," I said.

"He's very energetic," Wiggins told us. "Just this morning, he and I set out for a little walk."

At the word 'walk', the dog barked happily.

"When we set out, he immediately dashed off to the end of the road, then turned round and bounded back to me. He did this four times in total, in fact. After that, he settled down to match my speed, and we walked the remaining 81 feet to the end of the road at my pace. But it seems to me that if I tell you the distance from where we started to the end of the road, which is 625 feet, and that I was walking at four miles an hour, you ought to be able to work out how fast Rufus goes when he's running."

"Indeed we should," said Holmes, and turned to look at me expectantly.

What's the dog's running speed?

My Solution

I tried many approaches and finally resorted to a graphic one to see if I could guess the answer. But my drawing was not accurate enough. So I gave up and peeked at the answer (and not the solution). I adjusted my graphic to the correct answer and then tried to derive it from the diagram again, but still no luck. I finally resorted to a brute force approach that involved a lot of tricky arithmetic (ugh!) and numerous dead-end paths simplifying the expressions. But finally I got it.

Figure 1 shows a space-time diagram of the problem with Wiggins's world-line in green and Rufus's in brown. Wiggins's speed is v_W = 4 mph and Rufus's is an unknown, but faster, speed v_R.

There are fours segments with similar properties, namely, when Wiggins travels a distance D in time T, Rufus travels that distance D plus twice the remaining distance to the end of the road in the same time T. This yields a succession of equations as follows.

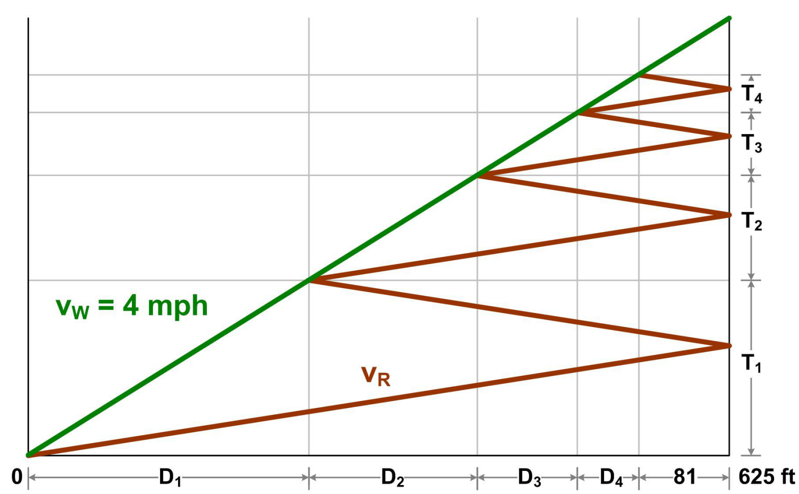

Figure 1 Space-time Diagram and Labels

$$\frac{D_1}{v_W} = T_1 = \frac{D_1 + 2(625 - D_1)}{v_R}$$

If we let $r = v_R/v_W$, then we get

$$\boxed{D_1 = \frac{2 \cdot 625}{1 + r}} \tag{1}$$

Next we get

$$rD_2 = D_2 + 2\big(625 - (D_1 + D_2)\big) = -D_2 + 2 \cdot 625\left(1 - \frac{2}{1 + r}\right) = -D_2 + 2 \cdot 625\left(\frac{r-1}{r+1}\right)$$

Therefore,

$$\boxed{D_2 = 2 \cdot 625 \frac{(r-1)}{(r+1)^2}} \tag{2}$$

Next we have

$$rD_3 = D_3 + 2\big(625 - (D_1 + D_2 + D_3)\big)$$

which eventually yields

$$\boxed{D_3 = 2 \cdot 625 \frac{(r-1)^2}{(r+1)^3}} \tag{3}$$

and similarly, skipping some agonizing computations, we get

$$\boxed{D_4 = 2 \cdot 625 \frac{(r-1)^3}{(r+1)^4}} \tag{4}$$

Adding up all the Ds ($D_1 + D_2 + D_3 + D_4 = 625 - 81$) and letting $x = (r - 1)/(r + 1)$ (so $r = (1 + x)/(1 - x)$), gives us

$$625 - 81 = \frac{2 \cdot 625}{r+1}\left[1 + x + x^2 + x^3\right] = 2 \cdot 625\left(\frac{1-x}{2}\right)\left[\frac{1-x^4}{1-x}\right] = 625(1 - x^4)$$

which yields $$x^4 = \frac{625 - (625 - 81)}{625} = 1 - \left(1 - \frac{81}{625}\right) = \frac{3^4}{5^4} \tag{5}$$

so $x = 3/5$. Solving for $r = v_R/v_W$, we get

$r = (1 + x)/1 - x) = (8/5)/(2/5) = 4.$

Thus Rufus runs 4 times faster than Wiggins walks, so Rufus's speed is 16 mph.

Figure 2 shows a graphic visualization of the solution. It was this graphic I tried to use to solve the problem, but in vain.

Comment. After the fact, I noticed that equations (1)-(4) show a

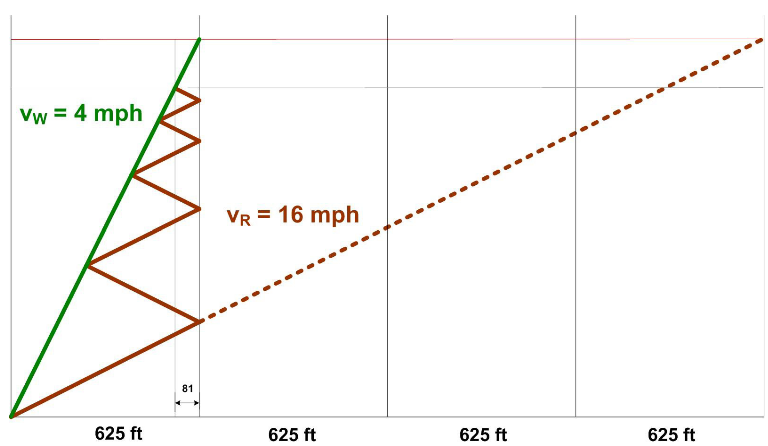

Figure 2 Solution Visualized

pattern of

$$D_{k+1} = xD_k = \frac{r-1}{r+1} D_k = \frac{v_R - v_W}{v_R + v_W} D_k \tag{6}$$

There is probably something there that would reveal a faster way to arrive at the answer, but I have spent enough time on the problem for now.

Dr. Watson's Solution

After completing the argument above, I finally had a look at Dr. Watson's solution:

The answer is 16 mph. The overall distance to the end of the road in feet is $625 = 5^4$, and the end of the dog's running time is when the distance in feet is $81 = 3^4$. These quad roots are obviously in the ratio of 5:3, so the sum of the two speeds and the difference of the two speeds must be in the ratio of 5:3 [why?], and thus the two speeds in the ratio of 4:1. Wiggins walks at 4 mph, so the dog runs at 16 mph.

I knew that ratio in equation (6) would be significant. However, I confess I don't follow Dr. Watson's reasoning. My equation (5) is essentially the majority of Dr. Watson's statement. I am not sure how one could jump to that equation without doing the intermediate math. Dr. Watson's simple answer feels a little bit like arguing after the fact. He must have known that this problem was difficult, since he put it in his "fiendish" collection.

References

[1] Dedopulos, Tim, *The Sherlock Holmes Puzzle Collection: The Lost Cases*, Metro Books, Sterling Publishing Co., New York, Carlton Books Ltd., London, 2015.