

Family Values

13 April 2020

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Here is a collection of puzzles from the great logic puzzle master Raymond Smullyan in a “Brain Bogglers” column for the 1996 *Discover* magazine ([1]).

1. **ELDON WHITE HAS FOUR DOGS.** One day he put out a bowl of dog biscuits. The eldest dog came first and ate half the biscuits plus one more. Then the next dog came and ate half of what he found plus one more. Then the next one came and ate half of what she found plus one more. Then the little one came and ate half of what she found and one more, and that finished the biscuits. How many biscuits were originally in the bowl?
2. Eldon once bought a very remarkable plant, which, on the first day, increased its height by a half, on the second day by a third, on the third day by a quarter, and so on. How many days did it take to grow to 100 times its original height?
3. In addition to four dogs, Eldon has four children. The youngest, Betty, is nine years old; then there are twin boys, Arthur and Robert; and finally there’s Laura, the eldest, whose age is equal to the combined ages of Betty and Arthur. Also, the combined ages of the twins are the same as the combined ages of the youngest and the eldest. How old is each child?
4. “How about a riddle?” asked Robert. “Very well,” said Eldon. “What is it that is larger than the universe, the dead eat it, and if the living eat it, they die?”



My Solutions

1. This problem is reminiscent of the 1770 Card Game Problem,¹ which also was a Brain Bogglers contribution (by a different author). First, let B_n be the number of biscuits the n th dog found in the bowl. Then we have

$$B_{n+1} = \frac{1}{2} B_n - 1$$

for $n = 1, 2, 3, 4$, where “ B_5 ” represents the bowl after the fourth dog ate, namely, $B_5 = 0$. So the idea is to work backwards:

$$B_{n-1} = 2(B_n + 1)$$

So

$$B_4 = 2(0 + 1) = 2$$

$$B_3 = 2(2 + 1) = 6$$

$$B_2 = 2(6 + 1) = 14$$

$$B_1 = 2(14 + 1) = 30$$

Therefore the first dog found the full bowl of **30 biscuits**.

2. I confess I misread this problem at first and thought we had to estimate the growth of the partial

¹ <http://josmfs.net/2019/10/20/1770-card-game-problem/>

sums for the harmonic infinite series. That seemed too difficult for the supposed audience of these problems. A glance at the answer showed me my error.

Let x_n be the height of the plant on day n , where day 0 represents the initial height of the plant x_0 . Then on the first day,

$$x_1 = x_0 + \frac{1}{2} x_0 = (1 + \frac{1}{2}) x_0 = (\frac{3}{2}) x_0$$

on the second day,

$$x_2 = x_1 + \frac{1}{3} x_1 = (1 + \frac{1}{3}) x_1 = (\frac{4}{3}) x_1 = (\frac{4}{3})(\frac{3}{2}) x_0 = (\frac{4}{2}) x_0$$

on the third day,

$$x_3 = x_2 + \frac{1}{4} x_2 = (1 + \frac{1}{4}) x_2 = (\frac{5}{4}) x_2 = (\frac{5}{4})(\frac{4}{3}) x_1 = (\frac{5}{4})(\frac{4}{3})(\frac{3}{2}) x_0 = (\frac{5}{2}) x_0$$

So on the n th day,

$$x_n = x_{n-1} + \frac{1}{n+1} x_{n-1} = (1 + \frac{1}{n+1}) x_{n-1} = (\frac{n+2}{n+1}) x_{n-1}$$

Therefore

$$x_n = (\frac{n+2}{n+1}) x_{n-1} = (\frac{n+2}{n+1})(\frac{n+1}{n}) x_{n-2} = (\frac{n+2}{n}) x_{n-2} = (\frac{n+2}{n-1}) x_{n-3} = \dots = (\frac{n+2}{2}) x_0$$

We want to know the day when $\frac{n+2}{2} = 100$. That is simply $n = 2 \cdot 100 - 2 = 198$.

3. Let B, A, R, and L be the ages of the four children Betty, Arthur, Robert, and Laura, respectively. Then

$$B = 9, A = R, L = B + A, \text{ and } A + R = B + L$$

So

$$L = 9 + A = 2A - 9$$

or

$$A = 18 = R \Rightarrow L = 27.$$

4. I confess I did not get this. So Smullyan's solution will have to be the final word.

Brain Bogglers (Smullyan's) Solutions

1. This problem is best solved by working it backward. How many biscuits did the fourth dog find? Well, dividing that number by 2 and subtracting 1, we get 0. Reversing the procedure, adding 1 to 0 and then multiplying by 2, we get 2.

Thus the last dog found 2 biscuits. Adding 1 and multiplying by 2, we get 6, the number of biscuits found by the third dog. Adding 1 and multiplying by 2, we get 14, the number found by the second dog. Finally, adding 1 and multiplying by 2, we get 30, the number of biscuits originally in the bowl.

2. The plant was initially, say, 1 unit tall (the length of a unit really doesn't matter). After one day the plant was $1\frac{1}{2}$ units tall. The next day it gained $\frac{1}{3}$ of $1\frac{1}{2}$, which is $\frac{1}{2}$, so it was then 2 units tall. The day after that, it gained $\frac{1}{4}$ of 2, which is again $\frac{1}{2}$. So the plant actually gained $\frac{1}{2}$ unit each day. After 198 days it gained 99 units, and it was then 100 times its height on the first day. Thus the answer is 198 days.
3. Let x be the age of each twin and y be the age of Laura. Then $y = x + 9$ and $y + 9 = 2x$. This makes $x = 18$ and $y = 27$. Thus the twins are 18 and Laura 27.
4. Nothing is larger than the universe. The dead eat nothing. If the living eat nothing, they die. So the answer is *nothing*.

References

- [1] Smullyan, Raymond, "Brain Bogglers: Family Values," *Discover*, Walt Disney Publ. Group Inc., New York, November 1996, p.128

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