The Track Problem

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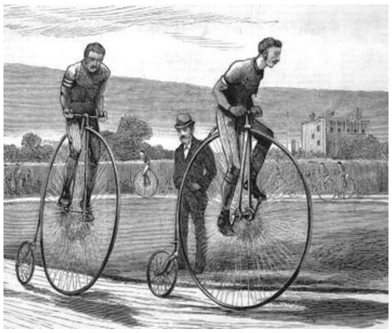

Again we have a puzzle from the Sherlock Holmes puzzle book by Dr. Watson (aka Tim Dedopulos) ([1] p.61).

Our pursuit of the dubious Alan Grey, whom we encountered during *The Adventure of the Third Carriage,* led Holmes and myself to a circular running track where, as the sun fell, we witnessed a race using bicycles. There was some sort of substantial wager involved in the matter, as I recall, and the track had been closed off specially for the occasion. This was insufficient to prevent our ingress, obviously.

One of the competitors was wearing red, and the other blue. We never did discover their names. As the race started, red immediately pulled ahead. A few moments later, Holmes observed that if they maintained their pace, red would complete a lap in four minutes, whilst blue would complete one in seven.

Having made that pronouncement, he turned to me. “How long would it be before red passed blue if they kept those rates up, old chap?”

Whilst I wrestled with the answer, Holmes went back to watching the proceedings. Can you find the solution?

My Solution

Figure 1 shows the problem setup via a space-time diagram. The red racer has begun a second lap before the blue racer has finished his first. But the red racer does not overtake the blue racer until blue’s second lap and the red racer’s third lap.

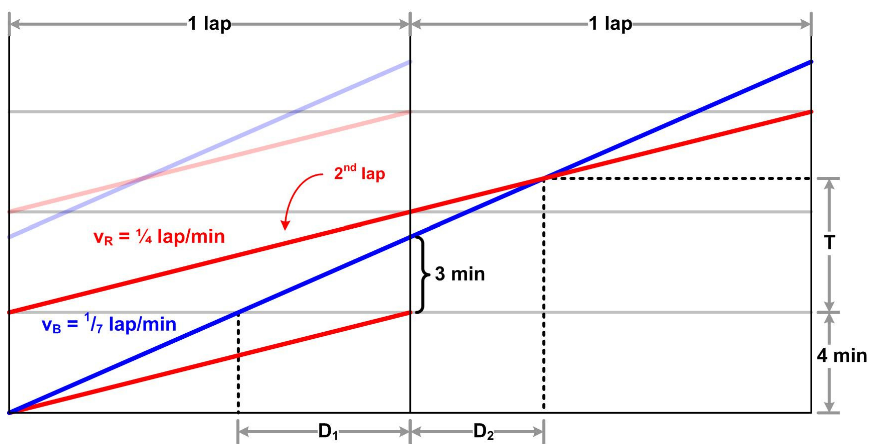

Figure 1 A Space-time Diagram of the Race

Let v_R = ¼ lap/min and $v_B = {}^1/_7$ lap/min be the speeds of the red and blue racers, respectively. Let T be the time until the rendezvous after the red racer has completed his first lap (in 4 minutes). Let D_1 be the distance the blue racer has yet to complete of his first lap after the red racer completes his first lap, namely, in the last 3 minutes of his 7 minute time to complete a lap. And let D_2 be the distance the blue racer has to cover in his second lap (or the red racer in his third lap) until the rendezvous.

Now consider what happens when the red racer begins his second lap after 4 minutes.

$$v_R\, T = 1 + D_2 \quad \Rightarrow \quad \tfrac{1}{4}\, T = 1 + D_2$$

and noting that $D_1 = v_B \times 3 = {}^3/_7$ of a lap,

$$v_B\,T = D_1 + D_2 \quad \Rightarrow \quad {}^1\!/_7\,T = {}^3\!/_7 + D_2$$

Subtracting these two equations yields

$${}^3\!/_{28}\,T = {}^4\!/_7 \quad \Rightarrow \quad T = 5\ {}^1\!/_3 \text{ minutes}$$

So adding the time of 4 minutes for the first lap, we get the total time until the red racer passes the blue racer is

$$T = 9 \text{ min } 20 \text{ sec}$$

Dr. Watson's Solution

The answer is 9 minutes and 20 seconds. Blue is moving 4/7 the speed of red, and red needs to have run blue's distance plus one whole lap in order to pass him. After one lap of red's, blue is 3/7 of a lap behind. After two, he's 6/7 down. It should be obvious that red is closing the distance at exactly 3/7ths of a lap for each lap of his own. He has 1/7th to go, so that will take him a third of a lap. So the total distance is 2 and 1/3rd laps, which at 4 minutes a lap is 9 minutes and 20 seconds.

References

[1] Dedopulos, Tim, *The Sherlock Holmes Puzzle Collection: The Lost Cases*, Metro Books, Sterling Publishing Co., New York, Carlton Books Ltd., London, 2015.