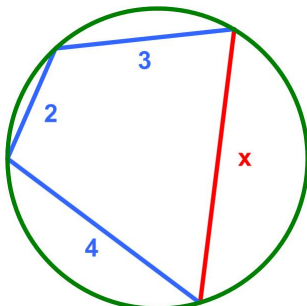


Quad in Circle Problem

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Here is another Brain Bogglers problem from 1987 by Michael Stueben ([1]).



A quadrilateral with sides three, two, and four units in length is inscribed in a circle of diameter five. What's the length of the fourth side of the quadrilateral?

Like a number of other Brain Bogglers this problem also uses an insight that makes the solution easy.

Solution

The trick is to transform the problem to an equivalent one that is easy to solve. Consider an axis of rotation as the perpendicular bisector of the line joining the points of the quadrilateral shown in Figure 1. Now rotate the upper triangle about this axis, yielding a new quadrilateral inscribed in the circle with the same edges but in a different order. The length of the edges is preserved since the rotation is a rigid motion.

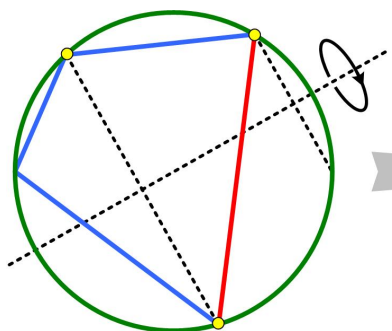


Figure 1 Rotation of Two Sides of Quadrilateral

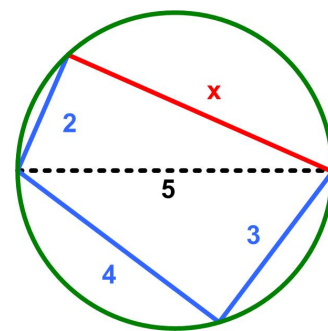
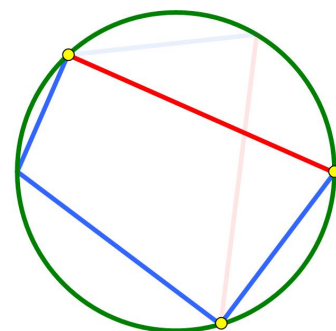


Figure 2 Solution

Figure 2 shows that with the circle diameter of 5, we have a 3-4-5 right triangle. Similarly, any triangle inscribed in a circle with one side a diameter must be a right triangle. Therefore, using the Pythagorean Theorem we have $x^2 = 25 - 4 = 21$, so $x = \sqrt{21}$.

(There probably needs to be some argument to claim no other triangle with legs 3 and 4 can be inscribed in the circle with diameter 5 unless it is the right triangle with hypotenuse 5, namely the diameter.)

References

[1] Stueben, Michael, "Brain Bogglers," *Discover*, March 1987

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