New Years Sum

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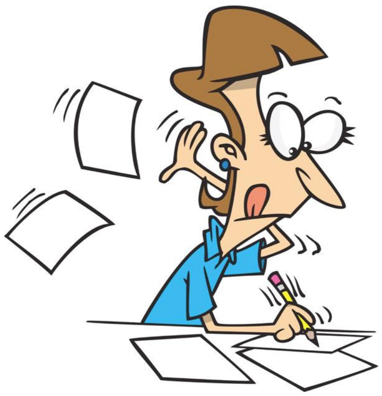

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Here is another problem from the 2020 Math Calendar to stimulate your mind ([1]).

$$9+6+4+\frac{8}{3}+\frac{16}{9}+\ldots=?$$

Remember that the answers to Math Calendar problems must all be whole numbers representing days of the month.

Solution

First, write out the series in terms of powers of 2 and 3.

$$S=3^2+2\cdot 3+2^2+\frac{2^3}{3^1}+\frac{2^4}{3^2}+\ldots+\frac{2^{n+2}}{3^n}+\ldots$$

Complete the pattern in the first three terms.

$$S=\frac{2^0}{3^{-2}}+\frac{2^1}{3^{-1}}+\frac{2^2}{3^0}+\frac{2^3}{3^1}+\frac{2^4}{3^2}+\ldots+\frac{2^{n+2}}{3^n}+\ldots$$

Factor out 2^2 and then $(2/3)^{-2}$ to reveal a geometric series.

$$S=2^2\frac{2^{-2}}{3^{-2}}+2^2\frac{2^{-1}}{3^{-1}}+2^2\frac{2^0}{3^0}+2^2\frac{2^1}{3^1}+2^2\frac{2^2}{3^2}+\ldots+2^2\left(\frac{2}{3}\right)^n+\ldots$$

$$S=2^2\frac{2^{-2}}{3^{-2}}\left(1+\frac{2}{3}+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+\ldots+\left(\frac{2}{3}\right)^{n+2}+\ldots\right)$$

Thus we have the answer.

$$S=9\frac{1}{1-\frac{2}{3}}=27$$

References

[1] Rapoport, Rebecca and Dean Chung, *Mathematics 2020: Your Daily epsilon of Math*, Point Rock, Quarto Publishing Group, New York, 2020. January