The Bicycle Problem

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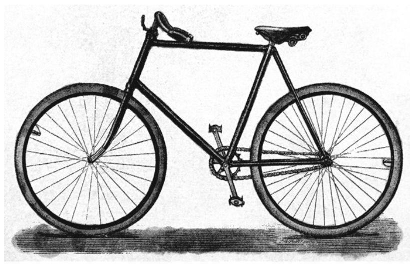

A fun, relatively new, Sherlock Holmes puzzle book by Dr. Watson (aka Tim Dedopulos) has puzzles couched in terms of the Holmes-Watson banter. The following problem ([1] p.113) is a variation on the Sam Loyd Tandem Bicycle Puzzle.[1]

I noticed Holmes looking distracted one morning over breakfast, tossing a piece of toast into tile air before catching it again, over and over like a cat with a toy.

"Something on your mind, old man?" I asked him.

"The valencies of sulphur," he replied. "Particularly in the way that they relate to its propensity to form astringents with zinc."

"Ah," I replied.

"Here's something mostly unrelated for you to chew over, my dear Watson. Say you and I have a single bicycle between us, and no other transport options save walking. We want to get the both of us to a location eighteen miles distant as swiftly as possible. If my walking speed is five miles per hour compared to your four, but for some reason—perhaps a bad ligament—my cycling speed is eight miles per hour compared to your ten. How would you get us simultaneously to our destination with maximum rapidity?"

"A cab," I suggested.

"Without cheating," Holmes replied, and went back to tossing his toast in the air.

My Solution

Figure 1 shows a space-time diagram of the problem setup and some plausible solutions. The solid lines represent riding the bicycle and the dotted lines walking. Watson's world-line is red and Sherlock Holmes's is blue. We begin by assuming Sherlock rides and Watson walks. The light lines

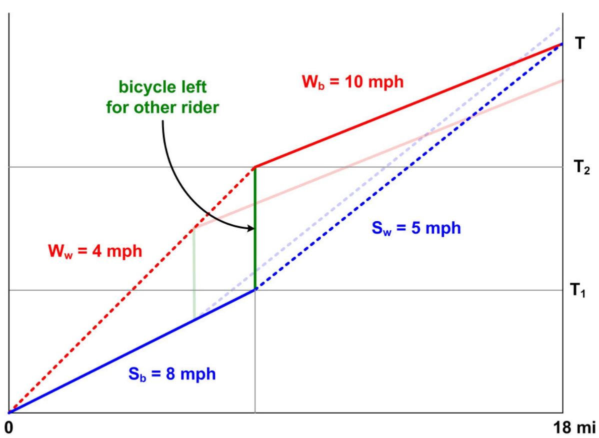

Figure 1 Problem Setup

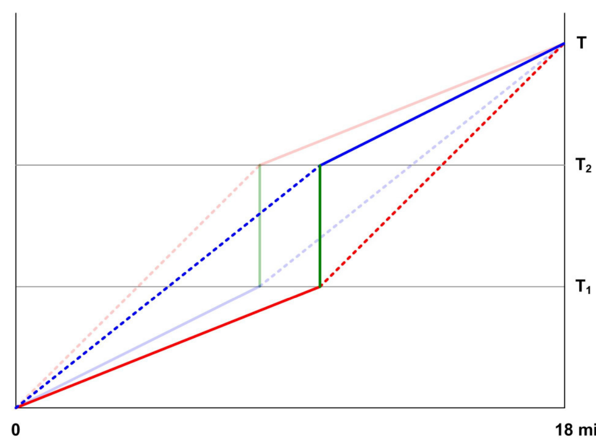

Figure 2 Alternative Solution

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[1] http://josmfs.net/2019/07/24/tandem-bicycle-puzzle/

show what happens if Sherlock leaves the bicycle too soon and walks longer: he arrives later than Watson. By riding the bicycle longer he will shorten his over-all time (and lengthen Watson's). Clearly the optimal situation, yielding the least time, is when both Watson and Holmes arrive at the same time T.

Figure 2 shows it does not matter if Watson rides the bicycle first; the minimal time is the same. This is because the original and alternative world-lines of each person form parallelograms.

From Figure 1 we see that the distance traveled when Sherlock leaves the bicycle is the same as the distance walked when Watson arrives at the bicycle. So

$$S_b T_1 = W_w T_2 \quad \Rightarrow \quad 8 T_1 = 4 T_2 \quad \text{or} \quad T_2 = 2 T_1$$

And the remaining distance traveled by Watson on the bicycle is the same as walked by Sherlock, so

$$W_b (T - T_2) = S_w (T - T_1) \quad \Rightarrow \quad 10 (T - 2 T_1) = 5 (T - T_1) \quad \text{or} \quad T = 3 T_1$$

Therefore,

$$18 \text{ mi} = S_b T_1 + S_w (T - T_1) = 8 T_1 + 5 (3 T_1 - T_1) \quad \Rightarrow \quad T_1 = 1 \text{ hour}$$

and so the fastest trip takes

$$T = 3 \text{ hours}$$

Watson's Solution

The trick is to divide our journey by the ratio of our comparative speeds—5:4, in this instance. So as the faster walker and slower rider, Holmes would ride for 4/9$^{\text{ths}}$ of the way, and I, in the opposite situation, would ride 5/9$^{\text{ths}}$ of the way. If we each do our riding in one stint, it doesn't make any difference who goes first. Either Holmes could ride eight miles, then leave the bicycle for me to pick up, and walk; or I could ride ten miles, leaving the bicycle for Holmes to pick up. The trip will take 3 hours, with each of us riding for one hour and walking for two, and the bicycle waiting for an hour in the middle.

References

[1] Dedopulos, Tim, *The Sherlock Holmes Puzzle Collection: The Lost Cases*, Metro Books, Sterling Publishing Co., New York, Carlton Books Ltd., London, 2015.