# Threewise

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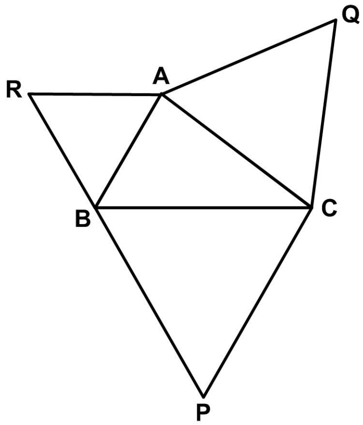


Here is another simple problem from *Futility Closet* ([1]).

Draw an arbitrary triangle [ABC] and build an equilateral triangle on each of its sides, as shown. Now show that [straight lines] AP = BQ = CR.

## My Solution

To be clear, Figure 1 shows explicitly the lines joining the vertices stated in the problem. Consider the upper two lines CR and BQ. They are part of congruent triangles ACR and BQA respectively, because side AC = side AQ (equilateral triangle), side AR = side AB (equilateral triangle), and angle RAC = angle BAQ (60° + common angle) (Figure 2). Therefore, CR = BQ. The other lines are handled similarly.

From the unambiguous diagram in the problem statement it appears that the three lines cross at a common point. I wonder how hard that is to prove.

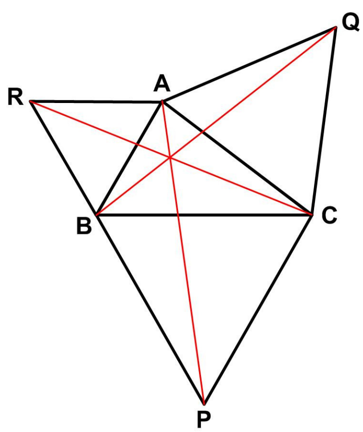


**Figure 1 Unambiguous Problem Statement**

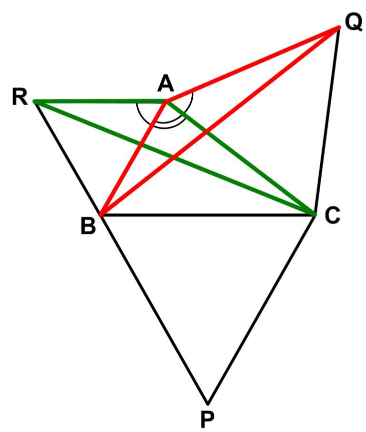


**Figure 2 Congruent Triangles**

## Futility Closet Solution

Rotating the figure 60° counterclockwise around B carries A to R and P to C. And rotating it 60° clockwise around A carries B to R and Q to C. So AP = BQ = CR.

From Edward Barbeau, Murray Klamkin, and William Moser, *Five Hundred Mathematical Challenges*, 1995.

This is essentially what the congruent argument I used implies, where in my case the rotation would be 60° counterclockwise around A and I picked different lines. I felt the Futility Closet solution was a bit obscure.

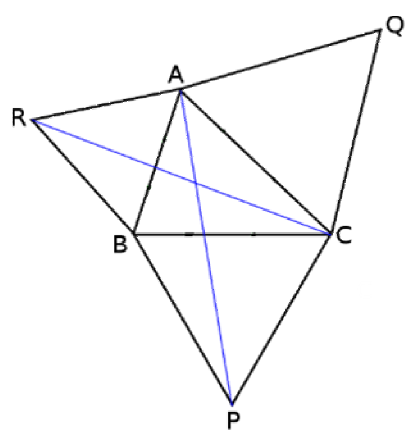


**Figure 3 Futility Closet Solution**

## References

[1] "Threewise" *Futility Closet*, 23 January 2015 (http://www.futilitycloset.com/2015/01/23/threewise/, retrieved 6/18/2015)